Phantom Cosmology, Future Singularities and Thermodynamics

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Outline

Abstract

Can we speak on thermodynamics in phantom cosmology?

The current observational data

Expansion

The current observational data indicates that the Universe is undergoing an accelerated expansion in recent times ($z \sim 0.45$). Observational evidence: SN Ia and CMBA, Large Scale Structure formation, Baryon Oscillations and Weak Lensing also suggest such an accelerated expansion of the Universe. **And**, an over-accelerated expansion **is not ruled out** from these observational evidence ($\omega < -1$, phantom!).

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z is the redshift parameter and $\omega = p/\rho$ is the equation of state parameter

One of the most challenging problems of modern cosmology is to identify the cause of this late time acceleration. Many theoretical approaches have been employed to explain the phenomenon of late time cosmic acceleration. A positive cosmological constant can lead to accelerated expansion of the universe **but** it is plagued by problems, for instance, the coincidence problem and the abundance of elements, in particular, the Litium.

Some possibilities

A theoretical answer, dark energy? We identify dark energy as a source of acceleration ($\ddot{a} > 0$). In particular, a first option is the cosmological constant (Λ). Other option is $\omega = \omega(z)$, reasonable option?

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- ΛCDM (Λ + cold dark matter ($\omega = 0$)) is a good model, but $\omega < -1$ (phantom dark energy)? And interacting cosmic fluids? There is a hard observational evidence for this. See:

Dark Matter and Dark Energy Interactions: Theoretical Challenges, Cosmological Implications and Observational Signatures, B. Wang, E. Abdalla, F. Atrio-Barandela and D. Pavón, Rept. Prog. Phys. ***79*** *(2016) no.9, 096901.*

A little pause...

Before to continue. Dark matter ($\longrightarrow \ddot{a} < 0$) and dark energy ($\longrightarrow \ddot{a} > 0$), a reflex of our ignorance. We do not know the intimate nature of these cosmic "components".

The observational data today

- $\omega(0) = -1.13^{+0.13}_{-0.10}$ (*Planck mission*),
- $\omega(0) = -1.166^{+0.072}_{-0.069}$
  (*Pan − STARRS*1 *Medium Deep Survey*; *SN Ia*),
- $\omega(0) = -1.155^{+0.076}_{-0.080}$ (*SN Ia*).

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We observe a certain "preference" for $\omega(0) < -1$ (**phantom!**).
Either way, the observation will have the last word.

Dynamics

According to GR + FLRW (cosmological principle), we have (with $k = 0$ and without cosmological constant)

$$3H^2 = \rho,$$

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{1}{6}\left(\rho + 3p\right),$$

$$\dot{\rho} + 3H\left(\rho + p\right) = 0.$$

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- FLRW is: $ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$,
- $1 + z = a(0)/a(z)$, being a the cosmic scalar factor.

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Flat case $k = 0$

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$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{1}{2}\left(\omega + \frac{1}{3}\right)\rho,$$
$$\dot{\rho} + 3H\rho(1+\omega) = 0.$$

- We note that $\omega + 1/3 > 0 \longrightarrow \ddot{a} < 0$ and $\omega + 1/3 < 0 \longrightarrow \ddot{a} > 0$. In the literature $-1 < \omega < -1/3$: quintessence zone.
- **The phantom divide is** $\omega = -1$**.** In this case $\rho = \text{const.} \longrightarrow p = -\rho$ and $H(t) = H(0)\exp\left(\sqrt{\rho/3}t\right)$: de Sitter.
- **The phantom zone is** $\omega < -1$.

Examples

$\omega = 1$ (stiff matter), $\omega = 1/3$ (radiation), $\omega = 0$ (dust). Usually, $\omega = 0$ is called dark matter. These cosmic components lead to $\ddot{a} < 0$.

Future Singularities

We consider $\omega < -1$. Then,

$$\dot{H} + H^2 = \frac{3}{2}\left(|\omega| - \frac{1}{3}\right) H^2 \longrightarrow H(t) = \frac{2}{3\left(|\omega| - 1\right)} \left(t_s - t\right)^{-1},$$

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In this case we have a Type I-singularity: **Big Rip**:
$\rho(t \longrightarrow t_s) \longrightarrow \infty$, $|p(t \longrightarrow t_s)| \longrightarrow \infty$, $a(t \longrightarrow t_s) \longrightarrow \infty$.
Here, the Hubble parameter and its cosmic time derivative diverge.

Other singularities

- Type II, **Sudden**: $\rho(t \longrightarrow t_s) \longrightarrow \rho_s$, $|p(t \longrightarrow t_s)| \longrightarrow \infty$, $a(t \longrightarrow t_s) \longrightarrow a_s$,
- Type III, $\rho(t \longrightarrow t_s) \longrightarrow \infty$, $|p(t \longrightarrow t_s)| \longrightarrow \infty$, $a(t \longrightarrow t_s) \longrightarrow a_s$,
- Type IV, $\rho(t \longrightarrow t_s) \longrightarrow 0$, $|p(t \longrightarrow t_s)| \longrightarrow 0$, $a(t \longrightarrow t_s) \longrightarrow a_s$ and higher derivatives of H diverge.

S. Nojiri, S. D. Odintsov and S. Tsujikawa, Phys. Rev. D**71**, 063004 (2005).

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- Type IV, $\rho\left(t\longrightarrow t_s\right)\longrightarrow 0$, $\left|p\left(t\longrightarrow t_s\right)\right|\longrightarrow 0$, $a\left(t\longrightarrow t_s\right)\longrightarrow a_s$ and higher derivatives of H diverge.

S. Nojiri, S. D. Odintsov and S. Tsujikawa, Phys. Rev. D**71**, 063004 (2005).

A feature of a **Big Rip** is that all bound-state systems disintegrate before the final singularity. In the ΛCDM model, there is no such divergence and no disintegration because the dark energy density remains constant.

By kicking the singularities towards infinity

Little Rip

This case corresponds to an **abrupt event**, i.e., it is not strictly speaking a future space-time singularity, as it takes place at an infinite cosmic time. The cosmic scale factor, the Hubble parameter and its cosmic time derivative all diverge at an infinite cosmic time. **Little Rip** models, despite not having a final singularity, also produce the disintegration of bound structures. A **Little Rip** interpolates between these two cases, the **Big Rip** and the ΛCDM model as its boundaries.

From

$$3H^2 = \rho,$$
$$\dot{\rho} + 3H(\rho + p) = 0,$$

and by using $p = -\rho - f(\rho)$, with $f(\rho) > 0$, we obtain

$$a = a_0 \exp\left(\frac{1}{3}\int \frac{d\rho}{f(\rho)}\right),$$

and then

$$t = \frac{1}{\sqrt{3}}\int \frac{d\rho}{\sqrt{\rho}f(\rho)},$$

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where the condition for a **Big Rip** singularity is that the last integral converges. By taking $f(\rho) = A\rho^{\alpha}$, we have:

$$\rho(t) = \left[\sqrt{3}A(1/2-\alpha)\right]^{1/(1/2-\alpha)} t^{1/(1/2-\alpha)},$$

and a future singularity, at finite time, can be avoided if $\alpha < 1/2$. For instance, if we consider $\alpha = 1$

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$$\rho(t) = \rho_0 \left(\frac{3AH_0}{2}\right)^{-2} (t_s - t)^{-2},$$

where $t_s = t_0 + \left(\frac{2}{3A}\right) H_0^{-1}$. For $\alpha = 1/2$

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$$\rho=\exp\left(\sqrt{3}At\right).$$

Little Sibling of the Big Rip

This behaviour again corresponds to an **abrupt event** rather than a future space-time singularity. The presence of such event in the future of the Universe leads in a finite time to a dissociation of the local structure of the Universe, which begins by ripping apart the large scale structures, such as clusters of galaxies, and only later on affecting structures of the size of the Solar system.

At this event, the Hubble rate and the scale factor blow up but the cosmic derivative of the Hubble rate does not. Consequently, this abrupt event takes place at an infinite cosmic time where the scalar curvature diverges.

From

$$3H^2 = \rho + \Lambda,$$
$$\dot{\rho} + 3H(\rho + p) = 0,$$

with $p = -\rho - A/3$, $A > 0$ and $p = \omega\rho$, we write

$$(1 + \omega)\rho = -\frac{A}{3} \Longrightarrow \omega = -1 - \frac{A}{3\rho}.$$

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As the energy density gets infinitely large in the future, the equation of state parameter ω behaves as the one of a cosmological constant.

From the conservation equation we obtain

$$\rho(a) = \Lambda + A \ln\left(\frac{a}{a_0}\right) \Longrightarrow \rho(a \longrightarrow \infty) \longrightarrow \infty$$

$$\Longrightarrow H(a \longrightarrow \infty) \longrightarrow \infty \text{ and } \omega(a \longrightarrow \infty) \longrightarrow -1.$$

The solution for the cosmic scale factor is:

$$a(t) = a_0 \exp\left(\frac{3}{A}\left[\left(\frac{A}{6}(t - t_0) + \sqrt{\frac{\Lambda}{3}}\right)^2 - \frac{\Lambda}{3}\right]\right),$$

and the universe is not asymptotically de Sitter. If we consider $t \longrightarrow \infty$, we have

$$a(t \longrightarrow \infty) \longrightarrow a_0 \exp\left(\tfrac{A}{12} t^2\right),\ \Lambda - \text{independent!},$$

and $\dot{H} = A/6 = \text{const}$, but $R = 6\left(2H^2 + \dot{H}\right)$ diverges. In de Sitter: $\rho(a) = \Lambda$, $H(a) = \sqrt{\Lambda/3}$, $a(t) = a_0 \exp\left(\sqrt{\Lambda/3}\,t\right)$ and $R = \text{const}$. $(H(a) = \sqrt{\Lambda/3})$.

More explicitly, **Little Sibling of the Big Rip** versus Λ

$$\frac{H_{LSBR}\left(a\right)}{\sqrt{\Lambda/3}} = \sqrt{1 + \left(\frac{A}{\Lambda}\right)\ln\left(a/a_0\right)} = 1 + ...$$

I. Albarran, M. Bouhmadi-López, F. Cabral and P. Martín-Moruno, JCAP **1511** (2015) no.11, 044.

So, only the first one, **Big Rip**, is regarded as a true singularity since it occurs at a finite cosmic time. For this reason, we refer to the others as **abrupt events**

Viscous (bulk viscosity) future singularities

These are **new** future singularities obtained by incorporating bulk viscosity:

- $\rho(t \longrightarrow t_s) \longrightarrow \infty$, $|p(t \longrightarrow t_s)| \longrightarrow 0$, $a(t \longrightarrow t_s) \longrightarrow \infty$, and
- $\rho(t \longrightarrow t_s) \longrightarrow \infty$, $|p(t \longrightarrow t_s)| \longrightarrow \text{const.}$, $a(t \longrightarrow t_s) \longrightarrow \infty$,

and the Hubble parameter and its cosmic time derivative all diverge at a finite cosmic time (t_s).

N. Cruz, **S. Lepe** and F. Peña, Phys. Lett. B **646** (2007) 177-182; M. Cataldo, N. Cruz and **S. Lepe**, Phys. Lett. B **619**, 5 (2005).

Viscous schemes

The idea is

$$p \longrightarrow p_{eff} = p + \Pi = \omega\rho + \Pi = \omega_{eff}\rho,$$

where Π is the bulk viscous pressure and $\omega_{eff} = \omega + \Pi/\rho$. And, given that $\Pi < 0$, we can have $\omega_{eff} < 0$ even if $\omega > 0$ and so, a sort of "dark energy" which produce a positive acceleration ($\ddot{a} > 0$).

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- **Question**, viscous dark matter as an (viable) alternative to dark energy? See, for instance:

Crossing the phantom divide with dissipative normal matter in the Israel–Stewart formalism, N. Cruz and **S. Lepe**, Phys. Lett. B**767** (2017) 103-109.

There are two main approaches for the bulk viscosity:

- Eckart formalism (non-causal), here $\Pi = -3\xi\left(\rho\right)H$ and $\xi\left(\rho\right) > 0$ (consequence of thermodynamics) is the bulk viscosity coefficient.
- Israel-Stewart formalism (causal) and non-linear extensions. Here, Π satisfies an differential equation.

An important detail (cited from arXiv:1612.07705): *the local equilibrium does not mean near equilibrium as variations of macroscopical physical quantities can be big over large distances and long time periods, and in fact includes most far-from-equilibrium situations observed in nature.*

Viscous Little Rip

I. Brevik, E. Elizalde, S. Nojiri and S. D. Odintsov, Phys. Rev. D**84** (2011) 103508.

Here, the key idea is

$$p_{eff} = -\rho - f(\rho) - 3\xi(\rho) H,$$

i. e., **Little Rip** plus Π. For instance, if $\xi(\rho) = \text{const.} = \xi_0$ and $f(\rho) = A\sqrt{\rho}$, we have

$$\rho(t) = \left[\left(\frac{\xi_0}{A} + \sqrt{\rho(0)}\right)\exp\left(\sqrt{\frac{3}{4}}At\right) - \frac{\xi_0}{A}\right]^2 .$$

Thus $\rho \longrightarrow \infty$ can be reached, but the universe needs an infinite time to do so.

Despite the earlier observations that viscosity basically supports the **Big Rip** singularity, it is also able to give rise to a non-singular, **Little Rip** cosmology, which is considered to be a viable alternative to ΛCDM cosmology.
And **do not be scared!** By considering the following **Little Rip** model

$$H(t) = H(0) \exp\left(\frac{A}{\sqrt{2}} t\right),$$

where $2.74 * 10^{-3} \left[Gyr^{-1}\right] \leq A \leq 9.67 * 10^{-3} \left[Gyr^{-1}\right]$ (Supernova Cosmology Project observational data), the time from present until the destruction of the Earth-Sun system is $\sim 8\,[Tyrs]$. And then, **take advantage of having fun!**

Related papers

- *Phantom Cosmology without* Big Rip *Singularity*, A. V. Astashenok, S. Nojiri, S. D. Odintsov and A. V. Yurov, Phys. Lett. B**709** (2012) 396-403.
- *Holographic dark energy in the DGP model,* N. Cruz, **S. Lepe** and F. Peña, Eur. Phys. J. C**72** (2012) 2162. In the framework of DGP model, the phantom state is a transitory one and the future evolution is de Sitter.
- *Cosmological future singularities in interacting dark energy models,* J. Beltrán, D. Rubiera-Garcia, D. Sáez-Gómez and V. Salzano, Phys. Rev. D**94** (2016) no.12, 123520.
- *Crossing the phantom divide with dissipative normal matter in the Israel-Stewart formalism,* N. Cruz and **S. Lepe**, Phys. Lett. B**767** (2017) 103-109.
- *Phantom solution in a non-linear Israel-Stewart theory,* M. Cruz, N. Cruz and **S. Lepe**, Phys. Lett. B**769** (2017) 159-165.
- *Dynamics of viscous cosmologies in the full Israel-Stewart formalism,* **S. Lepe**, G. Otálora and J. Saavedra, Phys. Rev. D**96** (2017) no.2, 023536.

Scalar field approach

Under this scheme:

$$p = \frac{1}{2}\dot{\phi}^2 - U(\phi),$$
$$\rho = \frac{1}{2}\dot{\phi}^2 + U(\phi) > 0 \text{(weak condition)}.$$

By thinking in $p = \omega\rho$, we write

$$\omega = \frac{\dot{\phi}^2/2 - U(\phi)}{\dot{\phi}^2/2 + U(\phi)},$$

and

$$\dot{\phi}^2/2 \gg U(\phi) \longrightarrow \omega \approx 1, \text{stiff matter},$$
$$U(\phi) \gg \dot{\phi}^2/2 \longrightarrow \omega \approx -1, \text{cosmological constant},$$

i. e., $-1 \leq \omega \leq 1$. But, how we can generate $\omega < -1$? Answer, $\dot{\phi}^2/2 < 0$, that is, a kinetic term with a "wrong" sign!
Many problems here, but is an interesting approach.

The transition (?) $\omega > -1 \longrightarrow \omega = -1 \longrightarrow \omega < -1$ and the zero temperature barrier

From the Gibbs law:

$$T\left(\frac{\partial p}{\partial T}\right)_n = \rho + p - n\left(\frac{\partial \rho}{\partial n}\right)_T ,$$

by using $p = \omega\rho$

$$T\frac{\partial}{\partial T}\left[\omega(z)\rho(z)\right]_n = \left[\omega(z)+1\right]\rho(z) - n(z)\left(\frac{\partial \rho}{\partial n}\right)_T ,$$

and $\dot{\rho} + 3H(\rho + p) = 0$, with $\rho = \rho(n, T)$, we obtain

$$T(z) = T(0)\left(\frac{1+\omega(z)}{1+\omega(0)}\right)\exp\left(3\int_0^z \omega(z)\, d\ln(1+z)\right).$$

It is evident that $1 + \omega(0)$ can not be independent of $1 + \omega(z)$! Additionally, "intelligent life" does not seem to live at zero temperature!

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1. What happen if we have a $\omega(z)$-model where we can have a continuos transition $\omega(z) > -1 \longrightarrow \omega(z) < -1$? In this case, with fixed-$1+\omega(0)$, at some time we would have a sign change in the temperature and so going by $T=0$! And then, how should we understand the access or escape from this zero-temperature? A system at zero temperature will continue at zero temperature regardless if we add an arbitrary amount of energy; a state at zero temperature does not interact with one non-zero temperature. This is an orthodox statement according to our reality.
2. Could we think in quantum fluctuations which could get us out of this state? In my opinion, it is still early use quantum mechanics to address this problem, we must understand better the thermodynamics in the cosmological field before making categorical statements. Nonetheless, see arXiv:1612.07705.

In brief, the cosmic evolution (described by the standard model) is: big-bang $\longrightarrow$inflation ($\omega \sim -1$)$\longrightarrow$ stiff matter ($\omega = 1$)$\longrightarrow$ radiation epoch ($\omega = 1/3$), structure formation (dark matter, $\omega \approx 0$, as dominant component), $\longrightarrow$ new cosmic acceleration (from around $z \sim 0.45$ and $\omega < -1/3$). The usual feeling (observational data) establishes that Λ ($\omega = -1$) is "omnipresent" through **all** the evolution. **But**, $\omega < -1$?

Usual cosmic components:

$$3H^2 = \Lambda + \rho_{st}(0)(1+z)^6 + \rho_{rad}(0)(1+z)^4 + \rho_{dm}(0)(1+z)^3 + \rho_{de}(z),$$

and $\rho_{de}(z)$ can be described by $\omega = -1$ (Λ), $-1 < \omega < -1/3$ (quintessence) or $\omega < -1$ (phantom dark energy).

Finally, can we speak on (standard) thermodynamics when phantoms are present?

Thermodynamics and phantom fields, **S. Lepe** and S. Odintsov, work in progress.

Thanks!