Neutron Stars in Horndeski gravity

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Outlook

- Horndeski theory
- Black hole solutions
- Neutron Stars
- Status of Horndeski gravity

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- Asymptotically locally AdS and flat black holes in Horndeski theory (10.1103/PhysRevD.89.084050)
- Asymptotically locally AdS and flat black holes in the presence of an electric field in the Horndeski scenario (10.1103/PhysRevD.89.084038)
- Neutron stars in general second order scalar-tensor theory: The case of nonminimal derivative coupling (10.1103/PhysRevD.92.044050)
- Slowly rotating neutron stars in the nonminimal derivative coupling sector of Horndeski gravity (10.1103/PhysRevD.93.084046)
- Axionic black holes in the K-essence sector of the Horndeski model (arXiv:1708.07194 [hep-th])

Horndeski gravity

Scalar-tensor theories

Which is the most general scalar-tensor theory we can construct with a single scalar field, a single metric tensor, in four dimensions and with a Levi-Civita connection?

Horndeski 40 years ago:

$$L = L(g_{\mu\nu}, g_{\mu\nu,j_1} \ldots g_{\mu\nu,j_1 \ldots j_p}, \phi, \phi_{,j_1}, \ldots \phi_{,j1 \ldots jq})$$

with $(p, q \geq 2)$. Constrained by the above considerations Horndeski demonstrated that the desired Lagrangian take the form

$L_H =$
$\kappa_1(\phi, X)\delta^{\alpha\beta\gamma}_{\mu\nu\rho}\nabla^\mu\nabla_\alpha\phi R^{\nu\rho}_{\beta\gamma} - \frac{4}{3}\kappa_{1,X}(\phi, X)\delta^{\alpha\beta\gamma}_{\mu\nu\rho}\nabla^\mu\nabla_\alpha\phi\nabla^\nu\nabla_\beta\phi\nabla^\rho\nabla_\gamma\phi$

+$\kappa_3(\phi, X)\delta^{\alpha\beta\gamma}_{\mu\nu\rho}\nabla_\alpha\phi\nabla^\mu\phi R^{\nu\rho}_{\beta\gamma} - 4\kappa_{3,X}(\phi, X)\delta^{\alpha\beta\gamma}_{\mu\nu\rho}\nabla_\alpha\phi\nabla^\mu\phi\nabla^\nu\nabla_\beta\phi\nabla^\rho\nabla_\gamma\phi$

[F$(\phi, X) + 2W(\phi)]\delta^{\alpha\beta}_{\mu\nu}R^{\alpha\beta}_{\mu\nu} - 4F(\phi, X)_{,X}\delta^{\alpha\beta}_{\mu\nu}\nabla_\alpha\phi\nabla^\mu\phi\nabla^\nu\nabla_\beta\phi$

-3[2F$(\phi, X)_{,\phi} + 4W(\phi)_{,\phi} + X\kappa_8(\phi, X)]\nabla_\mu\nabla^\mu\phi +$
$2\kappa_8(\phi, X)\delta^{\alpha\beta}_{\mu\nu}\nabla_\alpha\phi\nabla^\mu\phi\nabla^\nu\nabla_\beta\phi$

+$\kappa_9(\phi, X)$

Here:

- Covariant derivatives $\rightarrow$ Levi-Civita connection
- $X = -\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi$
- k_i are arbitrary functions of ϕ and X, F is function of the $k_i's$

Overlooked for a while, Horndeski theory reemerged in a more suitable way inspired by the generalization of the decoupling limit of the so called DGP model.
The DPG decoupling limit shows a very interesting local scalar field theory with the following main properties [arXiv:0005016, Dvali, Gabadadze and Porrati]

- Second order equations of motion

  $$\mathcal{L} \sim (\partial\phi)^2\Box\phi \rightarrow 3\Box\phi + \frac{1}{\Lambda^3}[(\Box\phi)^2 - (\partial_\mu\partial_\nu\phi)^2] = -\frac{1}{M_{pl}}T$$

- Invariant under

  $$\phi \rightarrow \phi + \phi_0 + b_\mu x^\mu$$

- Containing a self-screening mechanism (Vainshtein mechanism)

Motivated by this result, the most general scalar theory shearing the same properties of the DGP decoupling limit, namely Galileon theory (d=4) was constructed [arXiv:0811.2197, Nicolis, Ratazzi and Trincherini]:

$$
\begin{aligned}
L_2 &= (\partial\phi)^2 \\
L_3 &= (\partial\phi)^2(\Box\phi) \\
L_4 &= (\partial\phi)^2[(\Box\phi)^2 - (\partial_\mu\partial_\nu\phi)^2] \\
L_5 &= (\partial\phi)^2[(\Box\phi)^3 + 2(\partial_\mu\partial_\nu\phi)^3 - 3\Box\phi(\partial_\mu\partial_\nu\phi)^2].
\end{aligned}
$$

Soon after, it was demonstrated that its naive covariantization

$$
\begin{aligned}
\partial_\mu &\to \nabla_\mu \\
\eta_{\mu\nu} &\to g_{\mu\nu}
\end{aligned}
$$

do not produce second order EOM.

To obtain second order EOM we need

- Nonminimal couplings with the curvature [arXiv:0901.1314, Deffayet, Esposito-Farese and Vikman]

$$L = K(X) - G_3(X)\Box\phi + G_4(X)R + G_{4,X}(X)[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2]$$
$$+G_5(X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5,X}}{6}[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3]$$

- When G_i only depends on X this is the covariant version of Galileon gravity.
- A connection between the original theory and generalized Galileons was demonstrated [arXiv:1105.5723, Kobayashi, Yamaguchi and Yokoyama]
- Now sub-sectors of the theory are easy to be recognized BD theory, GR, K-essence theories, etc.

Black hole solutions in Horndeski theory

No-hair theorem for Galileons

A no-hair theorem was proposed for Galileon gravity in the context of spherically symmetric solutions [arXiv:1202.1296 Hui and Nicolis]. The argument is mostly based on the fact that this Lagrangian possesses the shift symmetry $\phi \to \phi + \phi_0$. Then

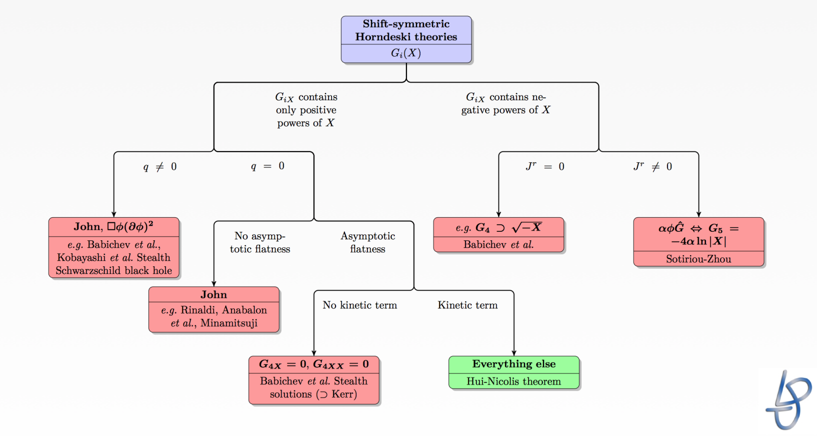
- $\nabla_\mu J^\mu = 0 \to J^r = \frac{c_0}{\sqrt{-g}}$
- Demanding a finite norm for the current on the horizon we must impose $c_0 = 0$
- Now, for theories in which L is at least of second order in ϕ' the current is always written as

$$J^r = \phi' \Theta(\phi', g, g', g'') = 0$$

For asymptotically flat solutions Θ approaches to a constant, then the only way to satisfy $J^r = 0$ is to have a trivial scalar field profile. Then the existence of scalar hair is ruled out.

[1604.06402 Babichev, Charmousis and Lehébel]

- $\phi(tr) = F(r)$
- $\phi(t,r) = Qt + F(r)$
- $\phi(x_i) = \lambda x_i$



Babichev-Charmousis solution

The equations of motion for our model read

$$G_{\mu\nu} + \Lambda g_{\mu\nu} + H_{\mu\nu} = 0\,, \nabla_\mu J^\mu = 0\,,$$

where

$$\begin{aligned}
H_{\mu\nu} \quad = \quad & -\frac{\alpha}{2\kappa}\left[\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}g_{\mu\nu}\nabla_\lambda\phi\nabla^\lambda\phi\right] \\
& -\frac{\eta}{2\kappa}\Big[\frac{1}{2}\nabla_\mu\phi\nabla_\nu\phi R - 2\nabla_\lambda\phi\nabla_{(\mu}\phi R^\lambda_{\nu)} \\
& -\nabla^\lambda\phi\nabla^\rho\phi R_{\mu\lambda\nu\rho} - (\nabla_\mu\nabla^\lambda\phi)(\nabla_\nu\nabla_\lambda\phi) \\
& +\frac{1}{2}g_{\mu\nu}(\nabla^\lambda\nabla^\rho\phi)(\nabla_\lambda\nabla_\rho\phi) - \frac{1}{2}g_{\mu\nu}(\Box\phi)^2 \\
& +(\nabla_\mu\nabla_\nu\phi)\Box\phi + \frac{1}{2}G_{\mu\nu}(\nabla\phi)^2 \\
& +g_{\mu\nu}\nabla_\lambda\phi\nabla_\rho\phi R^{\lambda\rho}\Big]\,, \\
J^\mu \quad = \quad & \left(\alpha g^{\mu\nu} - \eta G^{\mu\nu}\right)\nabla_\nu\phi
\end{aligned}$$

So let us consider the following static ansatz

$$ds^2 = -b(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2.$$

$$\phi(t,r) = Qt + F(r).$$

It is observed that $\nabla_\mu J^\mu = \nabla_t J^t + \nabla_r J^r = 0$. $J_t J^t$ vanishes on the horizon and then $J^r = 0$ solving the Einstein equation $\mathcal{E}_{tr}$.
In this way the scalar current is finite at the horizon and at the same time we have solve $\mathcal{E}_{tr}$.
Using the rest of the Einstein equations the system can be fully integrated.

A possible solution with all the parameters turned on is given by the following (A)dS metric

$$f = b = \frac{\alpha}{3\eta} r^2 + 1 - \frac{2M}{r},$$

$$F'(r) = \frac{Q}{f}\sqrt{1-f},$$

where the cosmological constant has to satisfy the following relation

$$\Lambda = -\frac{\alpha}{\eta}\left(1 - \frac{Q_p^2 \eta}{2\kappa}\right) .$$

and with a physical cosmological constant given by

$$\Lambda_m = -\frac{\alpha}{\eta}.$$

Other interesting solutions have been found for example in the following cases

- $\alpha = \Lambda = 0 \rightarrow$ Schwarzschild stealth.
- $Q = 0$, $\alpha \neq 0$ and $\Lambda \neq 0 \rightarrow$ asymptotically AdS BH's.

Axionic BH's in K-essence theory

$$\mathcal{S}[g,\phi] = \int [\kappa(R+2\Lambda) + \mathcal{L}(X_i)]\sqrt{-g}d^D x$$

where

$$\mathcal{L}(X_i) = -\sum_{i=1}^{2}\left(\frac{1}{2}\nabla^\mu\phi_i\nabla_\mu\phi_i + \gamma\left(\frac{1}{2}\nabla_i^\mu\nabla_\mu\phi_i\right)^k\right)$$

being $X_i = \frac{1}{2}\nabla^\mu\phi_i\nabla_\mu\phi_i$ with $i = 1, 2$. Einstein and KG equations read

$$\kappa(G_{\mu\nu}+\Lambda g_{\mu\nu}) = \frac{1}{2}\sum_i \left[\partial_\mu\phi_i\partial_\nu\phi_i - g_{\mu\nu}X_i + \gamma(kX_i^{k-1}\partial_\mu\phi_i\partial_\nu\phi_i - g_{\mu\nu}X_i^k)\right] \ ,$$

$$\left[(1+\gamma k X_i^{k-1})g^{\mu\nu} + \gamma k(k-1)X_i^{k-2}\nabla^\mu\phi_i\nabla^\nu\phi_i\right]\nabla_\mu\nabla_\nu\phi_i = 0.$$

Using the planar ansatz

$$ds^2 = -F(r)dt^2 + \frac{dr^2}{G(r)} + r^2 d\Sigma_{D-2}^2$$

we obtain the following black hole solution

$$F(r) = G(r) = \frac{r^2}{l^2} - \frac{2M}{r^{D-3}} - \frac{\lambda^2}{2(D-3)\kappa} - \gamma \frac{\lambda^{2k}}{2^k(2k+1-D)} r^{2(1-k)}$$

provided by $\phi_i = \lambda x_i$. To obtain physically relevant solutions we constraint k in such a way that the null energy condition holds

$$T_{\mu\nu} n^\mu n^\nu \geq 0\,, \quad i = 1, 2\,,$$

and that the sound speed is positive

$$c_s^2 = \frac{\mathcal{K},_{X_i}}{\mathcal{K},_{X_i} + 2X_i \mathcal{K},_{X_i,X_i}} > 0.$$

In our model a sufficient condition to satisfy simultaneously both requirements is $k > 1/2$. As we will discuss below, the further restriction $k > 3/2$ also guarantees that the solutions asymptotically match the GR ones and have finite ADM mass.

Static neutron stars in Horndeski theory

Some motivations

- Bulk properties of neutron stars can be used to constrain proposed equations of state that aim to describe the behavior of matter for densities greater than nuclear densities.
- Rotational instabilities can produce gravitational waves (Binary Black-Hole Merger GW150914)

Neutron stars

- The matter of the star is modeled by a perfect fluid. Observations of pulsar glitches show that departures from a perfect fluid in equilibrium are of order 10^{-5}
- Temperature of cold neutron stars do not affect their bulk properties. Can be assumed $0K$ because its termal energy ($<< 10MeV$, $10^{10}K$) is much smaller than fermi energies of the interior ($> 60MeV$).
- This allows us to use politropic equations of state (density-pressure relation)

$$\rho = \rho(P)$$

TOV equations

Now, in order to look for compact star configurations we need to construct the TOV system of equations. To do so we consider that the interior of our compact object will be given by a perfect fluid

$$T_{\mu\nu}^{(m)} = (\rho + P)u_\mu u_\nu + g_{\mu\nu} P$$

where the four velocity is given by $u_\mu = [-\sqrt{b(r)}, 0, 0, 0]$ such that $u^\mu u_\mu = -1$.

Then our field equations change in order to consider the effects of the fluid

$$\begin{aligned} G_{\mu\nu} + \Lambda g_{\mu\nu} + H_{\mu\nu} &= \frac{1}{2\kappa} T^{(m)}_{\mu\nu} \\ \nabla_\mu J^\mu &= 0 \\ \nabla_\mu T^{\mu\nu(m)} &= 0 \end{aligned}$$

In order to have asymptotically flat exterior solutions we will consider the specific case in which $\alpha = \Lambda = 0$. But before do so, let us check if this is also allowed from cosmological considerations.

The modify TOV equations for the case we are considering ($\alpha = \Lambda = 0$) are [arXiv:1504.05189, Cisterna, Delsate and Rinaldi]

$$\begin{aligned} rfb' &= (1-f)b \\ \eta f b F'^2 &= (1-f)\eta Q^2 + bPr^2 \\ Af' &= -B \end{aligned}$$

where

$$A = rbf^{-1}(Pr^2 + 4\kappa) - 3\eta Q^2 r\,,$$
$$B = 3(1-f)\eta Q^2 + bf^{-1}[6r^2 fP + (1+f)r^2\rho - 4\kappa(1-f)]$$

Constraining the parameters

Let us note that, before solve these equations we can impose some constraints in our parameters. Indeed, expanding around $r = 0$ with the regularity conditions $b(0) = b_0 > 0$, $b'(0) = 0$, $f(0) = 1$ and $P(0) = P_c$ we obtain

$$F'^2 = r^2 \left(\frac{P_c}{\eta} - \frac{2Q_0^2(3P_c + \rho_c)}{3\left(3Q_0^2\eta - 4\kappa\right)} \right) + \mathcal{O}(r^4),$$

$$P = P_c + \frac{(P_c + \rho_c)(3P_c + \rho_c)}{6\left(3Q_0^2\eta - 4\kappa\right)} r^2 + \mathcal{O}(r^4),$$

Immediately from these relations we observe that the value of ηQ_0^2 needs to be restricted in order to have a compact configuration provided by a real scalar field.

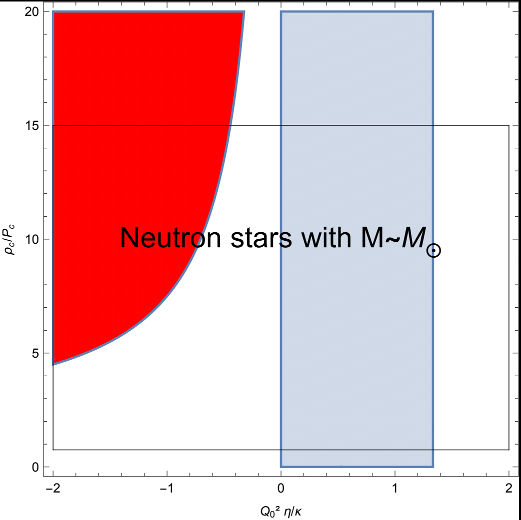
From the previous expansion we can show that in order to get compact configurations the second derivative of P must be negative ($P'' < 0$), otherwise P is a monotonic growing function, and compact configurations cannot exist. Then for $\eta > 0$

$$\eta Q_0^2 < \frac{1}{12\pi}$$

On the other hand for negative values of η compact configurations are not bounded, and they just satisfy $\eta < 0$.
At the same time we are looking for scalar field configurations that remain real, then further constraints need to be imposed. For positive values of η the first condition is still enough, nevertheless for η negative we need to satisfy

$$\frac{1}{4\pi} < |\eta| Q_0^2 \left(\frac{2\rho_c}{3P_c} - 1 \right)$$

Both results can be summarized in the following plot



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Once we have stablished these conditions we proceed to solve the TOV equations. To do so, we need to use an equation of state relating density and pressure. For simplicity we start considering the following polytrope adiabatic equation of state

$$P = K\rho_B^{1+1/n}$$

$$\rho = P + (P/K)^{n/(n+1)}$$

where ρ_B is the baryonic mass density, n is the polytropic index and K is a constant. We set $K = 123M_{\odot}^2$ and $n = 1$ since this model leads to compact objects with accepted mass and radius of neutron stars.

Solving the equations

Equipped with the equation of state we numerically solve our modified TOV equations in the following manner

- We chose $P(0) = P_c$ for a set of values of Q_0 and integrate the system as a Cauchy problem up to the star's surface located at $r = r_*$ defined as $P(r_*) = 0$.
- We use the boundary conditions $b_0 = 1$, $b_0' = 0$ and $P(0) = P_c$.
- We match the interior solutions to a Schwarzschild exterior solution and to a stealth scalar field.
- To compute the mass we solve

$$b(r_*) = b_\infty(1 - 2M/r_*), b'(r_*) = 2Mb_\infty/r_*^2$$

where M is the mass and b_∞ is a constant. In here we have redefined our time coordinate according to $t_p = t\sqrt{b_\infty}$ in order to match it with the time measured by a distant observe located at infinity. As a consequence we also redefine $Q_p = \frac{Q}{\sqrt{b_\infty}}$.

Results

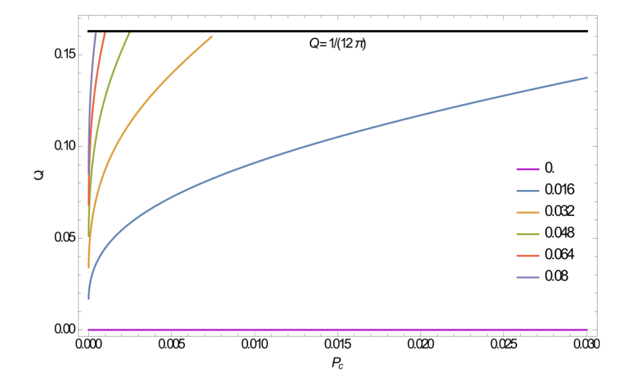


Figure: Lines of constant Q_p in the (P_c, Q) plan. The legend indicates the value of Q_p. The scalar field is no longer real for $Q > 1/(12\pi)$. All curves, except for $Q_p = 0$, reach a maximal value.

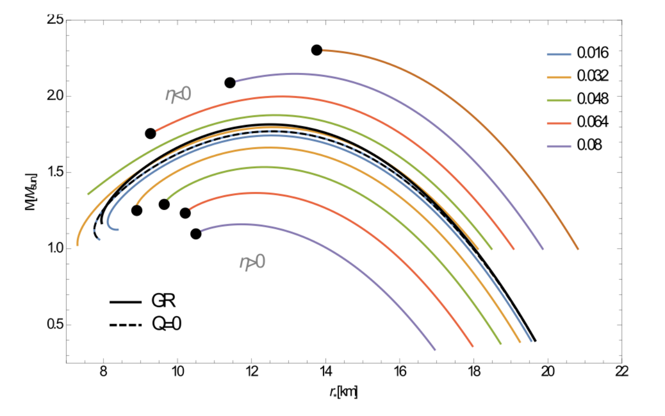


Figure: The mass-radius relation for various values of Q_p and $\eta = +1, -1$. The thick black curve is the GR prediction while the dashed one has $Q_p = 0$. The black dots are the points where the solutions cease to exist. Note that, for $\eta > 0$, the curves with $Q_p = 0.016$ and $Q_p = 0.032$ do not reach such a point in the chosen range of central pressures.

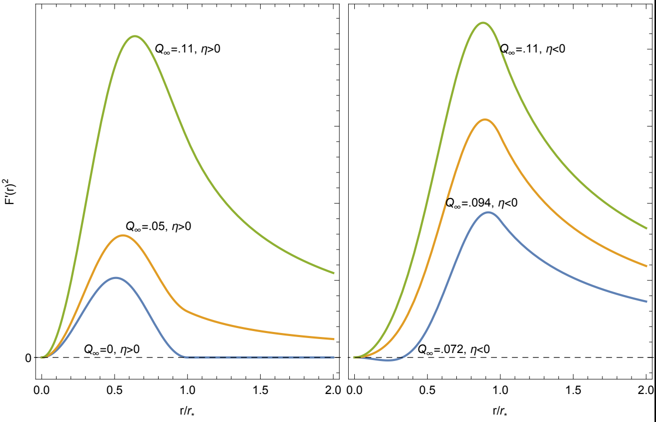


Figure: Plot of F'^2 for $\eta > 0$ (left) and $\eta < 0$ (right).

Slowly rotating neutron stars with nonminimal derivative coupling

There are two main effects that distinguish a rotating relativistic star from its static counterpart

- The shape of the star is flattened by centrifugal forces. This appears at least at second order in rotation.
- The local inertial frames are dragged by the rotation of the source generating the gravitational field.

At birth, neutron stars can rotate differentially, but once the star starts to cold several mechanism act to enforce uniform rotation

- Kinematical shear viscosity (years)
- Convective and turbulent motions (days)
- Superfluid phase transitions (minutes)

  This implies that the bulk features of an isolated rotating relativistic star can be modeled accurately by a rotating, zero-temperature perfect fluid.

Slowly rotating solutions

Let us consider the Hartle-Thorne slow-rotation approximation at first order in the parameter controlling rotation, ϵ. To do so we use the following ansatz

$$
\begin{aligned}
ds^2 &= -b(r)dt^2 + \frac{dr^2}{f(r)} + r^2\left[d\theta^2 + \sin^2\theta\left(d\varphi - \epsilon(\Omega_* - \omega(r)dt)\right)^2\right] \\
\Phi(t,r) &= Qt + F(r) + \epsilon\phi_1(t,r)
\end{aligned}
$$

where:

- $\omega(r) \rightarrow$ angular velocity acquired by an observer falling freely from infinity
- $\Omega_* \rightarrow$ constant angular velocity of a point of the fluid

One can prove that at first order in ϵ, black hole solutions ($\rho = P = 0$) get the same correction as in pure GR, this means that we have [1508.03044 Berti, Maselli et al, 1508.06413 Cisterna, Delsate et al.]

$$\omega(r) = \frac{2J}{r^3} \to \bar{\omega}(r) = \Omega_* - \omega(r) = \Omega_* \left(1 - \frac{2I}{r^3}\right)$$

where J and I are the angular momentum and the moment of inertia respectively. At first order the only equation that is modified is $\mathcal{E}_{t\varphi}$ from where we compute $\omega(r)$. Knowing this we look for interior solutions, using

$$u_a = (-\sqrt{b}, 0, 0, \Omega_* u_\varphi)$$

where

$$u_\varphi = \frac{2\sin^2\theta(\omega - \Omega_*)r^2\epsilon}{\Omega_*\sqrt{b}} + \mathcal{O}(\epsilon^2)$$

and considering the following tabulated equations of state

- $BSk14$, $BSk19$, $BSk20$, $BSk21$, $EOSL$ and $SLy4$

which cover a wide range of nuclear parameters.

Maximal mass of neutron stars

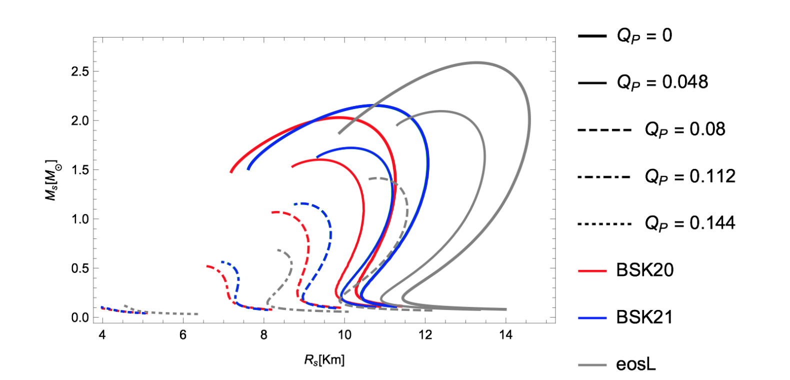
The most massive pulsars known to date are

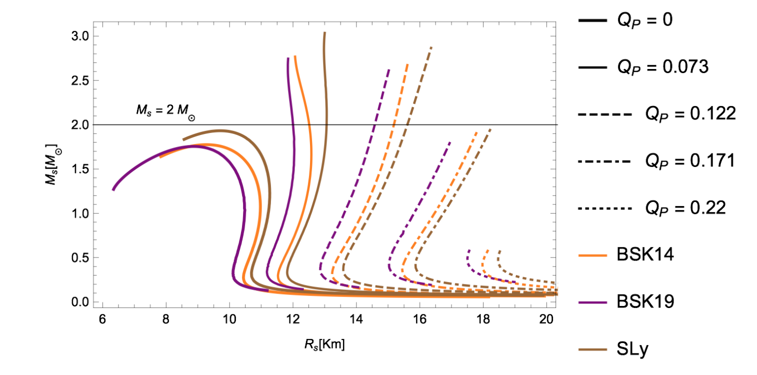
- $J1614 - 2230$ with $M = 1.97 \pm 0.04 M_{\odot}$.
- PSR J0348+0432 with a mass of $2.01 \pm 0.04 M_{\odot}$ and a relatively short orbital period: 2 hours and 27 minutes.
- Is our model supporting neutron star configurations with two solar masses mass?

- For the case $\eta > 0$, $BSk21$, $BSk20$ and $EOSL$ can reach $2M_\odot$ for some values of Q_p. The mass decreases when increasing the value of Q_p until the point in which the solution terminates once Q_p reaches its maximal allowed value.
- On the other hand, we observe that considering η negative the equations of state that were not reaching $2M_\odot$ can do it now increasing the value of Q_p.

These results can be summarized in the following figures:

[1602.06939 Cisterna, Delsate et. al]





3.0
2.5
2.0
1.5
1.0
0.5
0.0
6
8
10
12
14
16
18
20
$M_s = 2\,M_\odot$
$M_s[M_\odot]$
R_s[Km]
$Q_P = 0$
$Q_P = 0.073$
$Q_P = 0.122$
$Q_P = 0.171$
$Q_P = 0.22$
BSK14
BSK19
SLy

Inertia

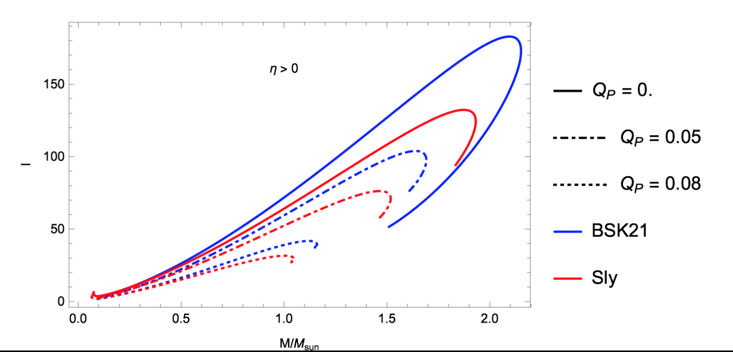


Figure: The mass - inertia curves for BSK21 and different values of Q_p with $\eta > 0$.

Inertia

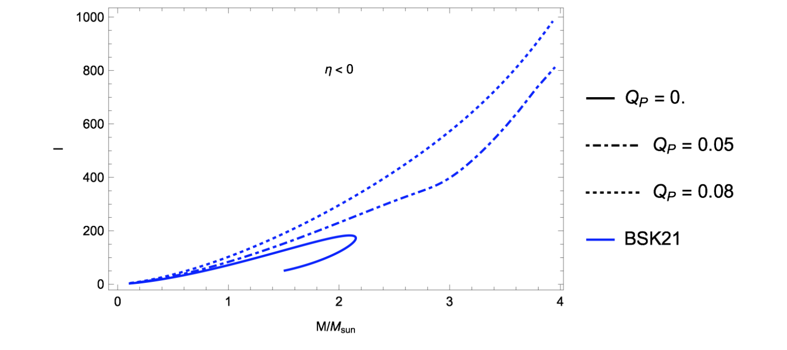


Figure: The mass - inertia curves for BSK21 and different values of Q_p with $\eta < 0$.

We obtain the maximal value of Q_p for which it is possible to construct neutron stars with masses of $2M_\odot$ for a given equation of state.

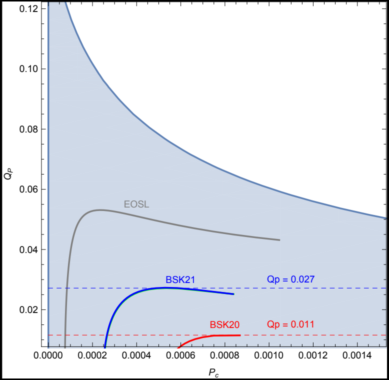


Figure: Constant mass curve with $M = 2M_\odot$ in the $P_c - Q_p$ plan. The shaded region is the region leading to compact solutions for all the EOS presented.

Status of Horndeski gravity

Gravitational waves in modified gravity

Let us consider a modified gravity theory in which we have extra degrees of freedom. The action for linearized gravitational waves reads

$$S_h = \frac{1}{2}\int d^3x dt\, M_*^2 \left[\dot{h}_A^2 - c_T^2(\nabla h_A)^2\right]$$

where M_* is an effective Mass Planck that would depend on the particular theory under consideration and h_A are the amplitudes of the polarization states of the perturbations h_{ab} around the Minkowski space. c_T is the speed of light which can be parametrized more convenient as

$$c_T^2 = 1 + \alpha_T$$

GW170817 and GRB 170817A

Ligo and Virgo for first time have simultaneously detected gravitational and electromagnetic waves coming from the same object:

- A NS merger located near NGC 4993.
- Using the measurements coming from the Gamma Ray Burst 170817A it was possible to constraint the speed of gravitational waves.
- It was stablished that gravitational waves practically travels at the speed of light $c_T^2 = 1 + \alpha_T$, with $|\alpha_T| \lesssim 1 \times 10^{-15}$

What about Horndeski theory?

On a Cosmological background Horndeski gravity gives [Baker et al. 1710.06394]

$$M_*^2\alpha_T \equiv 2X\left[2G_{4,X} - 2G_{5,\phi} - \left(\ddot{\phi} - \dot{\phi}H\right)G_{5,X}\right]$$

where $M_*^2 \equiv 2\left(G_4 - 2XG_{4,X} + XG_{5,\phi} - \dot{\phi}HXG_{5,X}\right)$.
In order to obtain $\alpha_T = 0$ we must precisely relate $G_{4,X}$, $G5$, X and $G_{5,\phi}$.

- Any non-trivial combination would also be sensitive to the matter content of the theory and small deviations from homogeneity and isotropy will produce high violations of these constraints.
- A more natural consideration implies that $G_{4,X} = 0$ and $G_5 = cte$. Then all terms that lead to nonminimal kinetic terms are ruled out, leaving Horndeski theory constructed only with

$$L = K(\phi, X) + G_3(\phi, X)\Box\phi + G_4(\phi)R$$

How to rescue the model?

The kind of model we are interested on are basically the sector of Horndeski theory that is ruled out by the recent detection of GW using neutron stars. Several models like beyond Horndeski , generalized Proca and Beyond generalized Proca suffers the same pathologies that the one mentioned before. Nevertheless if we still consider non-trivial $G_{4,5}$ one way to obtain $\alpha_T = 0$ is to consider the inclusion of terms belonging to the beyond Horndeski Lagrangian, this at the price of include higher order (ghost free) terms.

$$L_4^{bH} = F_4(X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma}{}_{\sigma\mu}\phi_\alpha\phi_{\nu\beta}\phi_{\rho\gamma}\phi$$

$$L_5^{bH} = F_5(X)\epsilon^{\mu\nu\rho\sigma}\epsilon_\mu^{\alpha\beta\gamma\delta}\phi_\alpha\phi_{\nu\beta}\phi_{\rho\gamma}\phi_{\sigma\delta}\phi$$

Conclusions

- Look for new black hole/stealth solutions in beyond Horndeski theory, the ones that allow cancelations of the extra contributions of Horndeski theory to the speed of gravitational waves.
- May Horndeski model rest in peace.

Gracias !