# Generalized equations of state and non singular universes

Seminario en Recuerdo del Dr. Sergio del Campo, Cofundador del grupo GACG

Felipe Contreras[1] Norman Cruz[2] Esteban González[2]

Departamento de Matemática, Facultad de Ciencia, Universidad de Santiago de Chile, Casilla 307, Santiago de Chile[1]

Departamento de Física, Facultad de Ciencia, Universidad de Santiago de Chile, Casilla 307, Santiago de Chile[2]

3 -5 de Diciembre, 2014

Pontificia Universidad Católica de Valparaíso

1. Introduction
   - Regular universes.
   - Main motivations for the study of EoS of the form:

     $$P = A\rho + \gamma\rho^{\beta} \quad ; A, \gamma, \rho \text{ constants}$$

   - Previous results corresponding to non singular universes: emergent and bouncing solutions.
2. Choosing an EoS
   - $P = -\rho - \gamma\rho^{\frac{1}{2}}$
   - Previous results.
3. Solutions for an emergent universe.
   - The scale factor and the Hubble parameter.
   - General properties.
4. Solution for a bouncing universe
   - The scale factor and the Hubble parameter.
   - General properties.
5. Discussion of our results.

# Emergent universes

$k > 0$ at early times allows Einstein static universes:

- no singularity
- no beginning of time
- no horizon problem
- inflation for at infinite past but a finite number of e-folds

$$a(t) = a_i \left[ 1 + \exp\left( \frac{\sqrt{2}t}{a_i} \right) \right]^{\frac{1}{2}}$$

[G.F.R. Ellis and R. Maartens, Class. Quantum. Grav. 21 (2004) 223 - 232]

[E.R. Harrison, Mon. Not. R. Astron. Soc 137 (1967) 69]

# Theoretical framework

1. Minimally coupled scalar field $\phi$ with a self interaction given by a special potential functions $V(\phi)$.
2. Generalized EoS.
3. Universes filled with normal matter and a phantom field.
   [U. Debuath, Class. Quantum. Grav. 25 (2008) 205019]
4. Self interacting Jordan-Brans-Dicke theory.
   [S. del Campo et al, JCAP 11 (2007) 030]
5. Brane world scenario.
   [A. Banerjee et al, Gen. Rel. Grav. 40 (2008) 1603]
6. Two measures field theories.
   [S. del Campo el at, JCAP 1006 (2010) 026]
7. Scalar field at false vacuum and tunneling.
   [P. Labaña, Phys. Rev. D86, 083524 (2012)]

# Early times

- $$P(\rho) = -\rho - \gamma\rho^{\lambda} \;;\lambda,\, \gamma \text{ constantes } (\gamma \neq 0)$$
  $$P(\rho) = (\gamma - 1)\rho \;;\text{for } \lambda = 1, \gamma < 0.$$

[J.D. Barrow, Phys. Lett. B 235, 40 (1990)]

Extend the range of known inflationary behaviors. Emergent solution but not discussed in this context. ($\lambda = \frac{1}{2}$, $\gamma < 0$)

UdeSantiago

- $$P(\rho) = -A\rho - \gamma\rho^{\frac{1}{2}} \quad ; A > -1, \gamma > 0$$

  Emergent solution:

  $$a(t) = a_0(\beta + \mathrm{e}^{\alpha t})^{\omega},$$

  with $\beta$, $a_0$ constants, $\alpha$ and $\omega$ in terms of $A$, $\gamma$.

  $$a(t \to -\infty) = a_0\beta\omega$$

  $$H(t \to -\infty) = \frac{\dot{a}}{a} = 0 \qquad \text{(space-time geodesically complete)}$$

  [Mukherjee et al, Class. Quantum Gavity. 23, 6927 (2006)]

# Late time behavior

- Observational data implies $\omega \approx -1$. $\omega > -1$ and $\omega < -1$ can not be discarded.

$$\begin{aligned} P(\rho) &= -\rho - A\rho^{\alpha} \\ a(t) &\sim a_0 e^{\mu e^{\xi t}} \qquad \mu,\ \xi \text{ constants} \end{aligned}$$

[H. Štefančic, Phys. Rev. D71, 084024 (2005)]

- Running vacuum energy in QFT is curved spacetime $(\Lambda(t), G(t))$

$$\omega(z) = -1 + \delta + f(z)$$

[S. Basilakos and J. Sola, Mon. Not. Roy. Astron. Soc. 437 (2014) 4, 331 - 3342]

# Choosing an EoS

We take the EoS:

$$P(\rho) = -\rho - \gamma \rho^{\frac{1}{2}}.$$

Flat universe described by a FRW metric and filled only with this fluid.

$$\begin{aligned} 3H^2 &= \rho, \\ \dot{\rho} + 3H(\rho + p) &= 0. \end{aligned}$$

Using the above equations. We obtain:

$$\frac{\ddot{a}}{a} - \frac{\dot{a}}{a} = \frac{\gamma\sqrt{3}}{2}.$$

Its solution takes the form

$$a(t) = a_0 \exp\left\{ \frac{2\rho_0^{\frac{1}{2}}}{3\gamma} \left[ \exp\left( \frac{\gamma\sqrt{3}}{2}(t - t_0) \right) - 1 \right] \right\},$$

with the initial condition $a(t_0) = a_0$ and $\rho(t_0) = \rho_0$

# Properties of the solutions

- $$a(t \to -\infty) = a_{min} = a_0 \exp\left(\frac{-2\rho_0^{\frac{1}{2}}}{3\gamma}\right)$$
- $H(t) \sim \exp\left(\frac{\gamma\sqrt{3}}{2}t\right)$, $\dot{H}(t) \sim \exp\left(\frac{\gamma\sqrt{3}}{2}t\right)$ and $\ddot{H}(t) \sim \exp\left(\frac{\gamma\sqrt{3}}{2}t\right)$
  Then $H$, $\dot{H}$, $\ddot{H} \to 0$ for $t \to -\infty$, which implies that the spacetime of the universe approaches a Minkowsky in the infinite past.

UdeSantiago

- $$\rho(a = a_{min}) = 0.$$
- The effective EoS is given by:

$$\omega_{eff} = -1 - \frac{\gamma}{\sqrt{\rho}}$$

$$t\omega(t \to -\infty) \to -\infty \implies \text{NEC is violated}$$

$$(\text{NEC} \iff (\rho + p \geq 0))$$

Since $\rho$ increases exponentially for positive times

$$\omega \to\sim -1$$

very fast, i.e., becomes a de Sitter like expantion.

- This solution can not be interpreted as the sum of other known fluids. For example for:

$$\rho = -A\rho - \gamma\rho^{\frac{1}{2}} \quad ; A > -1,$$

for some values of $A$:
dark energy, exotic matter and radiation
dark energy, exotic matter and cosmic strings
etc.

# The EoS and a phantom field

$$L = \int \sqrt{-g} \left( R - \frac{1}{2} \partial u \phi \partial^u \phi - V(\phi) \right) \mathrm{d}^4 x$$

with signature $(+,-,-,-)$.

$$\begin{aligned} \rho(\phi) &= -\dot{\phi}^2 + V(\phi) \\ P(\phi) &= -\dot{\phi}^2 - V(\phi) \end{aligned}$$

The solutions are:

$$V(\phi) = \frac{3\gamma^2}{256} \left[ 3(\phi - \alpha)^4 + 8(\phi - \alpha)^2 \right]$$

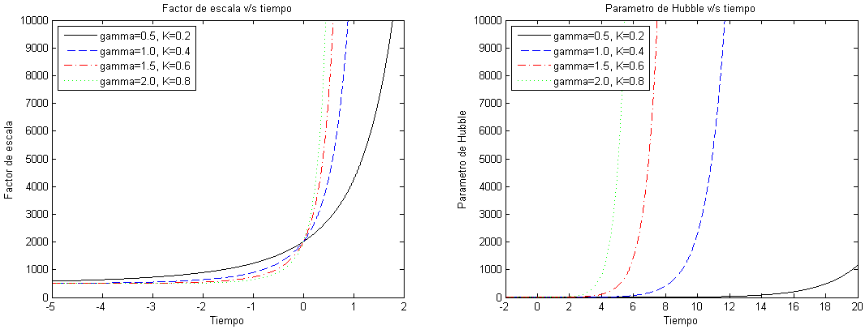
where $\alpha =$ constant of integration.

The field $\phi$ and $V$ as a function of time are given by:

$$\phi(t) = \frac{4}{\sqrt{3\gamma}}\rho_0^{\frac{1}{4}}\exp\left(\frac{\gamma\sqrt{3}}{4}(t-t_0)\right)+\alpha$$

$$V(t) = \rho_0\exp(\gamma\sqrt{3}(t-t_o))+\frac{\gamma}{2}\rho_0^{\frac{1}{2}}\exp\left(\frac{\gamma\sqrt{3}}{2}(t-t_0)\right)$$

For $t\rightarrow-\infty$, $\dot{\phi}\rightarrow 0$ and $V(t)\rightarrow 0$


Factor de escala v/s tiempo
Factor de escala
Tiempo
gamma=0.5, K=0.2
gamma=1.0, K=0.4
gamma=1.5, K=0.6
gamma=2.0, K=0.8
Parametro de Hubble v/s tiempo
Parametro de Hubble
Tiempo
gamma=0.5, K=0.2
gamma=1.0, K=0.4
gamma=1.5, K=0.6
gamma=2.0, K=0.8

# Solutions for a bouncing universe

Positive curvature $(k = +1)$ lads to bouncing solutions. For the EoS $P = A\rho - \gamma\rho^{\frac{1}{2}}$ we obtain a numerical solutions for the differential equations. $(A \geq -1)$

$$\begin{aligned} \frac{\mathrm{d}u}{\mathrm{d}t} &= \frac{\gamma\sqrt{3}}{2}\sqrt{u^2+k} + \frac{(u^2+k)}{a} \\ \frac{\mathrm{d}a}{\mathrm{d}t} &= u \end{aligned}$$

which leads to bouncing solutions.

# Exact solution

Fo the particular case

$$P(\rho) = -\frac{\rho}{3} - \gamma\rho^{\frac{1}{2}};$$

integration of Einstein's equations yields the following expression for the scale factor

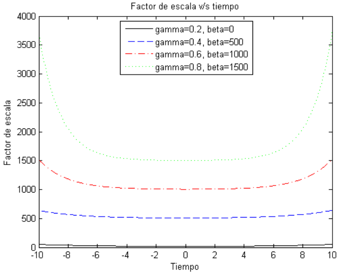
$$a(t) = \frac{2}{\gamma\sqrt{3}} \cosh\left(\frac{\gamma\sqrt{3}}{2}t + \alpha\right) + \beta,$$

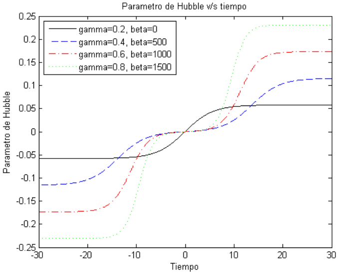
where $\alpha$, $\beta$ constants of integration.

The Hubble parameter takes the form:

$$H(t) = \frac{\sinh\left(\frac{\gamma\sqrt{3}}{2}t + \alpha\right)}{\frac{2}{\gamma\sqrt{3}}\cosh\left(\frac{\gamma\sqrt{3}}{2}t + \alpha\right) + \beta}$$

The behavior of $a(t)$ and $H(t)$ is showed in the following figures:




UdeSantiago

The energy density and the pressure in terms of the scale factor are given by:

$$\rho(a) = \left(\frac{3\gamma}{2} + \frac{\gamma}{2a}\right)^2,$$

$$P(a) = -\frac{1}{3}\left(\frac{3\gamma}{2} + \frac{\gamma}{2a}\right)^2 - \gamma\left(\frac{3\gamma}{2} - \frac{\gamma}{2a}\right).$$

Expanding both expressions, we obtain:

$$\rho(a) = \frac{9\gamma^2}{4} + \frac{3\gamma^2}{2a} + \frac{\gamma^2}{4a^2} = \rho_1 + \rho_2 + \rho_3,$$

$$P(a) = -\frac{9\gamma^2}{4} - \frac{\gamma^2}{a} - \frac{\gamma^2}{12a^2} = P_1 + P_2 + P_3.$$

So this EoS represents the sum of three know fluids .

Cosmological constant + quintessence + curvature
$(w = -1)$ $(w = -\frac{2}{3})$ $(w = -\frac{1}{3})$

# Discussion

- Generalized equations of state leads to new scenarios of emergent and bouncing universes.
- In some cases the choosing EoS can be interpreted as the sum of three known fluids.
- The matter source that leads to our emergent solution can be reinterpreted as that of a scalar phantom field with some potential.

# Work in progress

- To avoid phantom fields in the framework of scalar tensor theories.
- To investigate the inflationary behaviour of these solutions