Dynamical analysis of Generalized Galileon cosmology

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Outline

- Lovelock's theorem [Lovelock, 1971, Lovelock, 1972] limits the theories that one can construct from the metric tensor alone.
- Assuming that the metric tensor is the only field involved in the gravitational action. If the action can be written in terms of the metric tensor $g_{\mu\nu}$ alone, then we can write

$$S = \int d^4x \mathcal{L}(g_{\mu\nu}). \tag{1}$$

containing up to second derivatives of $g_{\mu\nu}$,
- Extremising it with respect to the metric gives the Euler-Lagrange expression

$$E^{\mu\nu}[\mathcal{L}] = \frac{d}{dx^\rho}\left[\frac{\partial \mathcal{L}}{\partial g_{\mu\nu,\rho}} - \frac{d}{dx^\lambda}\left(\frac{\partial \mathcal{L}}{\partial g_{\mu\nu,\rho\lambda}}\right)\right] - \frac{\partial \mathcal{L}}{\partial g_{\mu\nu}}, \tag{2}$$

and the Euler-Lagrange equation is $E^{\mu\nu}(\mathcal{L}) = 0$.

Theorem

(Lovelock's Theorem)
The only possible second-order Euler-Lagrange expression obtainable in a four dimensional space from a scalar density of the form $\mathcal{L} = \mathcal{L}(g_{\mu\nu})$ *is*

$$E^{\mu\nu} = \alpha\sqrt{-g}\left[R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\right] + \lambda\sqrt{-g}g^{\mu\nu}, \tag{3}$$

where α *and* λ *are constants, and* $R_{\mu\nu}$ *and* R *are the Ricci tensor and scalar curvature, respectively.*

- In $D \leq 4$ dimensions the more general action leading to Einstein equations are

$$\mathcal{L} = \alpha\sqrt{-g}R - 2\lambda\sqrt{-g} + \beta\epsilon^{\mu\nu\rho\lambda}R^{\alpha\beta}{}_{\mu\nu}R_{\alpha\beta\rho\lambda} + \gamma\sqrt{-g}\left(R^2 - 4R^{\mu}{}_{\nu}R^{\nu}{}_{\mu} + R^{\mu\nu}{}_{\rho\lambda}R^{\rho\lambda}{}_{\mu\nu}\right),$$

  where β and γ are also constants.

- The third and fourth terms in this expression do not, however, contribute to the Euler-Lagrange equations as

$$E^{\mu\nu}\left[\epsilon^{\alpha\beta\rho\lambda}R^{\gamma\delta}{}_{\alpha\beta}R_{\gamma\delta\rho\lambda}\right] = 0 \quad (4)$$

$$E^{\mu\nu}\left[\sqrt{-g}\left(R^2 - 4R^{\alpha}{}_{\beta}R^{\beta}{}_{\alpha} + R^{\alpha\beta}{}_{\rho\lambda}R^{\rho\lambda}{}_{\alpha\beta}\right)\right] = 0, \quad (5)$$

  where the action of $E^{\mu\nu}$ on any function X is defined as in Eq. (2). The first of these equations is valid in any number of dimensions, and the second is valid in four dimensions only.

Lovelock's theorem

Lovelock's theorem means that to construct metric theories of gravity with field equations that differ from those of General Relativity we must do one (or more) of the following:

- Consider other fields, beyond (or rather than) the metric tensor.
- Accept higher than second derivatives of the metric in the field equations.
- Work in a space with dimensionality different from four.
- Give up on either rank (2,0) tensor field equations, symmetry of the field equations under exchange of indices, or divergence-free field equations.
- Give up locality.

Aims

- Obtaining up to second order differential equations of motion
- Generalize quintessence/ K-essence models
- Obtaining field equations that are invariant under the Galilean symmetry $\phi \to \phi + c$, $\partial_\mu \phi \to \partial_\mu \phi + b_\mu$ in the limit of Minkowski spacetime, with c, b_μ constants.
- Obtaning an effective DE equation of the state parameter can lie in the quintessence or phantom regimes, or experience the phantom-divide crossing, without fine-tunning!

- The DGP cubic interaction

$$L = \Box\phi\partial_\mu\phi\partial^\mu\phi \tag{6}$$

This cubic interaction gives the second order e.o.m.

$$(\Box\phi)^2 - (\partial_\mu\partial_\nu\phi)(\partial^\mu\partial^\nu\phi) = 0 \tag{7}$$

- Galilean symmetry

$$\partial_\mu\phi \to \partial_\mu\phi + c_\mu \tag{8}$$

$$\delta L \propto c^\mu\Box\phi(\partial_\mu\phi)$$

$$= c^\mu\partial^\alpha\left[\partial_\alpha\phi\partial_\mu\phi - \frac{1}{2}\eta_{\alpha\mu}(\partial\phi)^2\right], \tag{9}$$

this is a total derivative.

Higher order terms (Nicolis, Rattazzi, Trincherini '09)

- Requirements:
  - The field equations are second order.
  - The terms are invariant up to total derivatives under the Galilean transformation

$$\partial_\mu \phi \to \partial_\mu \phi + c_\mu$$

- Schematically:
  - The Galileon terms also have a shift symmetry

$$\phi \to \phi + c$$

  - The Noether theorem implies that the e.o.m is written as total derivatives

$$\partial_\mu j^\mu = 0, \; j^\mu = -\frac{\partial L}{\partial(\partial_\mu \phi)}$$

  - In order for the e.o.m to be second order and satisfy the Galileon symmetry it is schematically given by

$$F\left[(\partial\partial\phi)\right].$$

The most general 4-dimensional scalar-tensor theories having second-order field equations are described by the Lagrangian [Felice & Tsujikawa, 2012]

$$\mathcal{L} = \sum_{i=2}^{5} \mathcal{L}_i \,, \tag{10}$$

where

$$\mathcal{L}_2 = K(\phi, X), \tag{11}$$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi, \tag{12}$$

$$\mathcal{L}_4 = G_4(\phi, X)\, R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)\,(\nabla^\mu\nabla^\nu\phi)\right], \tag{13}$$

$$\begin{aligned}\mathcal{L}_5 = {} & G_5(\phi, X)\, G_{\mu\nu}\,(\nabla^\mu\nabla^\nu\phi) \\ & - \frac{1}{6}\, G_{5,X}\left[(\Box\phi)^3 - 3(\Box\phi)\,(\nabla_\mu\nabla_\nu\phi)\,(\nabla^\mu\nabla^\nu\phi) + 2(\nabla^\mu\nabla_\alpha\phi)\,(\nabla^\alpha\nabla_\beta\phi)\,(\nabla^\beta\nabla_\mu\phi)\right].\end{aligned} \tag{14}$$

The total action is given by

$$S = \int d^4x\sqrt{-g}\,(\mathcal{L} + \mathcal{L}_m)\ , \tag{15}$$

where g is the determinant of the metric $g_{\mu\nu}$.

Imposing the flat Friedmann-Robertson-Walker (FRW) background metric we obtain,

$$\begin{aligned} 2XK_{,X} - K + 6X\dot{\phi}HG_{3,X} - 2XG_{3,\phi} - 6H^2G_4 + 24H^2X(G_{4,X} + XG_{4,XX}) \\ -12HX\dot{\phi}\,G_{4,\phi X} - 6H\dot{\phi}\,G_{4,\phi} + 2H^3X\dot{\phi}\,(5G_{5,X} + 2XG_{5,XX}) \\ -6H^2X\,(3G_{5,\phi} + 2XG_{5,\phi X}) = -\rho_m\,, \end{aligned} \tag{16}$$

$$\begin{aligned} K - 2X(G_{3,\phi} + \ddot{\phi}\,G_{3,X}) + 2(3H^2 + 2\dot{H})G_4 - 12H^2XG_{4,X} - 4H\dot{X}\,G_{4,X} \\ -8\dot{H}XG_{4,X} - 8HX\dot{X}\,G_{4,XX} + 2(\ddot{\phi} + 2H\dot{\phi})G_{4,\phi} + \\ +4XG_{4,\phi\phi} + 4X(\ddot{\phi} - 2H\dot{\phi})G_{4,\phi X} \\ -2X(2H^3\dot{\phi} + 2H\dot{H}\dot{\phi} + 3H^2\ddot{\phi})G_{5,X} - 4H^2X^2\ddot{\phi}\,G_{5,XX} + 4HX(\dot{X} - HX)G_{5,\phi X} \\ +2[2(\dot{H}X + H\dot{X}) + 3H^2X]G_{5,\phi} + 4HX\dot{\phi}\,G_{5,\phi\phi} = -p_m\,, \end{aligned} \tag{17}$$

Variation of (15) with respect to $\phi(t)$ provides its evolution equation

$$\frac{1}{a^3}\frac{d}{dt}\left(a^3 J\right) = P_\phi \,, \tag{18}$$

with

$$\begin{aligned} J &\equiv \dot{\phi}K_{,X} + 6HXG_{3,X} - 2\dot{\phi}\,G_{3,\phi} + 6H^2\dot{\phi}(G_{4,X} + 2XG_{4,XX}) - 12HXG_{4,\phi X} \\ &\quad +2H^3X(3G_{5,X} + 2XG_{5,XX}) - 6H^2\dot{\phi}(G_{5,\phi} + XG_{5,\phi X})\,, \qquad (19) \\ P_\phi &\equiv K_{,\phi} - 2X\left(G_{3,\phi\phi} + \ddot{\phi}\,G_{3,\phi X}\right) + 6(2H^2 + \dot{H})G_{4,\phi} + 6H(\dot{X} + 2HX)G_{4,\phi X} \\ &\quad -6H^2XG_{5,\phi\phi} + 2H^3X\dot{\phi}\,G_{5,\phi X}\,. \qquad (20) \end{aligned}$$

Finally, the evolution equation for matter takes the standard form

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0\,. \tag{21}$$

One class of Galileon scenarios has the above Lagrangian with the ansatzes:

$$K(\phi, X) = X - V(\phi), \;\; G_3(\phi, X) = -g(\phi)X, \;\; G_4(\phi, X) = \frac{1}{2}\frac{1}{8\pi G}, \;\; G_5(\phi, X) = 0, \quad (22)$$

corresponding to the action

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R - \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi) - \frac{1}{2}g(\phi)\partial^\mu\phi\partial_\mu\phi\Box\phi + \mathcal{L}_m\right]. \quad (23)$$

(one could straightforwardly include ansatzes with higher powers of X, such as the covariant Galileon model [Deffayet, Esposito-Farese, & Vikman, 2009], however for simplicity we remain to the above simple but non-trivial Galileon action).

Concerning $V(\phi)$ the usual assumption in dynamical investigations in the literature is to assume an exponential potential of the form

$$V(\phi) = V_0 e^{\lambda_V \phi}, \tag{24}$$

Concerning $g(\phi)$, and in order to remain general, we will consider two ansatzes, namely the exponential one

$$g(\phi) = g_0 e^{\lambda_g \phi}, \tag{25}$$

and the power-law one

$$g(\phi) = g_0 \phi^n. \tag{26}$$

The gravitational field equations (16) and (17) become

$$H^2 = \frac{8\pi G}{3}\Big(\rho_{DE} + \rho_m\Big), \tag{27}$$

$$\dot{H} = -4\pi G\Big(\rho_{DE} + p_{DE} + \rho_m + p_m\Big), \tag{28}$$

where we have defined the effective dark energy sector with energy density and pressure respectively:

$$\rho_{DE} = \frac{\dot{\phi}^2}{2}\Big(1 - 6gH\dot{\phi} + g_{,\phi}\dot{\phi}^2\Big) + V(\phi)\ , \tag{29}$$

$$p_{DE} = \frac{\dot{\phi}^2}{2}\Big(1 + 2g\ddot{\phi} + g_{,\phi}\dot{\phi}^2\Big) - V(\phi). \tag{30}$$

The scalar field equation (18) becomes

$$\ddot{\phi} + 3H\dot{\phi} + 2g_{,\phi}\dot{\phi}^2\ddot{\phi} + \frac{1}{2}g_{,\phi\phi}\dot{\phi}^4 - 3g\dot{H}\dot{\phi}^2 - 6gH\dot{\phi}\ddot{\phi} - 9gH^2\dot{\phi}^2 + V_{,\phi} = 0\ , \tag{31}$$

We can define the dark energy equation-of-state parameter as

$$w_{DE} \equiv \frac{p_{DE}}{\rho_{DE}} = \frac{\frac{\dot{\phi}^2}{2}\left(1 + 2g\ddot{\phi} + g_{,\phi}\dot{\phi}^2\right) - V(\phi)}{\frac{\dot{\phi}^2}{2}\left(1 - 6gH\dot{\phi} + g_{,\phi}\dot{\phi}^2\right) + V(\phi)}. \tag{32}$$

One can clearly see that in this scenario, according to the form of $g(\phi)$, w_{DE} can be quintessence-like, phantom-like, or experience the phantom divide crossing during the evolution!

For the above scenario to be free of ghosts and Laplacian instabilities, and thus cosmologically viable, two conditions must be satisfied [De Felice & Tsujikawa, 2010, Felice & Tsujikawa, 2012, Appleby & Linder, 2012]. In our case,

$$c_S^2 \equiv \frac{6w_1 H - 3w_1^2 - 6\dot{w}_1 - 6\rho_m}{4w_2 + 9w_1^2} \geq 0, \tag{33}$$

for the avoidance of Laplacian instabilities associated with the scalar field propagation speed, and

$$Q_S \equiv \frac{(4w_2 + 9w_1^2)}{3w_1^2} > 0\,, \tag{34}$$

for the absence of ghosts, where in our case

$$w_1 \equiv g\dot{\phi}^3 + 2H, \tag{35}$$

$$w_2 \equiv 3\dot{\phi}^2 \left[\frac{1}{2} + g_{,\phi}\dot{\phi}^2 - 6Hg\dot{\phi}\right] - 9H^2. \tag{36}$$

Finally, we stress that according to (32) and (33),(34) the phantom phase can be free of instabilities and thus cosmologically viable, as it was already shown for Galileon cosmology [Felice & Tsujikawa, 2012].

In the scenario at hand we introduce the auxiliary variables:

$$x = \frac{\kappa\dot{\phi}}{\sqrt{6}H},\ y = \frac{\kappa\sqrt{V(\phi)}}{\sqrt{3}H},\ z = g(\phi)H\dot{\phi},\ v = \frac{1}{\phi}. \tag{37}$$

Using these variables the Friedmann equation (27) becomes

$$(1-6z)x^2 + y^2 + \frac{\sqrt{6}zg'(\phi)x^3}{g(\phi)} + \frac{\rho_m}{3H^2} = 1. \tag{38}$$

Moreover, using (38) and (29) we can write the matter and dark energy density parameters as:

$$\Omega_m \equiv \frac{\rho_m}{3H^2} = 1 - \left[(1-6z)x^2 + y^2 + \frac{\sqrt{6}zg'(\phi)x^3}{g(\phi)}\right]$$

$$\Omega_{DE} \equiv \frac{\kappa^2\rho_{DE}}{3H^2} = (1-6z)x^2 + y^2 + \frac{\sqrt{6}zg'(\phi)x^3}{g(\phi)}. \tag{39}$$

Note that in the limit $g(\phi) \to 0$ the above quantities are well-defined, and they coincide with the usual quintessence ones [Copeland, Liddle & Wands, 1998].

Cosmological observables

Dark-energy equation-of-state parameter (32):

$$w_{DE} = \frac{6z^2\lambda_g^2x^4 + 3\sqrt{6}(1-2z)z\lambda_g x^3 + [3z(3z-4)+1]x^2 - \sqrt{6}y^2z(2\lambda_g+\lambda_V)x + y^2(6z-1)}{\left[zx^2\left(\sqrt{6}x\lambda_g - 6\right) + x^2 + y^2\right]\left[z\left(9zx^2 + 2\sqrt{6}\lambda_g x - 6\right)+1\right]}. \tag{40}$$

and the deceleration parameter:

$$\begin{aligned} q &\equiv -1 - \frac{\dot{H}}{H^2} = \frac{1}{2} + \frac{3}{2}w_{tot} \\ &= \left\{2z\left(9zx^2 + 2\sqrt{6}\lambda_g x - 6\right)+2\right\}^{-1}\left\{9\sqrt{6}(1-2z)z\lambda_g x^3 + [36(z-1)z+3]x^2 \right. \\ &\left. +18z^2\lambda_g^2x^4 + \sqrt{6}z\left[\left(2-6y^2\right)\lambda_g - 3y^2\lambda_V\right]x + \left(3y^2-1\right)(6z-1)\right\}. \end{aligned} \tag{41}$$

Finally, the two instability-related quantities:

$$\begin{aligned} c_S^2 = &\left\{x\left[z\left(9zx^2 + 2\sqrt{6}\lambda_g x - 6\right)+1\right]^2\right\}^{-1}\left\{3\sqrt{6}z^3\lambda_g x^4 + 3z^2x^3\left(2\lambda_g^2 - 6z + 5\right)\right. \\ &\left. -27z^4x^5 + 2\sqrt{6}(1-4z)z\lambda_g x^2 + \left\{z\left[3z\left(5-3y^2\right)-4\right]+1\right\}x + \sqrt{6}y^2z\lambda_V\right\} \end{aligned} \tag{42}$$

and

$$Q_S = \frac{3x^2\left[z\left(9zx^2 + 2\sqrt{6}\lambda_g x - 6\right)+1\right]}{(3zx^2+1)^2}. \tag{43}$$

Late-time attractors for Scenario 1: Exponential potential and exponential coupling function.

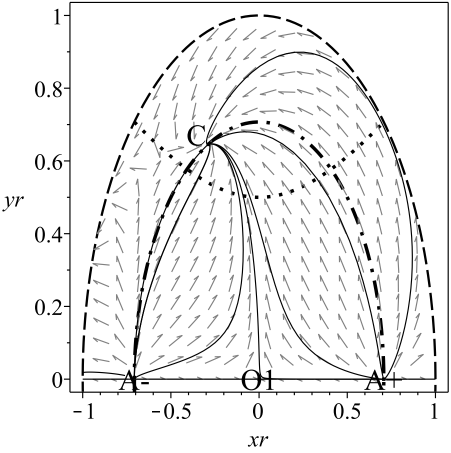
Cr. P.	x_c	y_c	z_c	Exist for
C	$-\frac{\lambda_V}{\sqrt{6}}$	$\sqrt{1-\frac{\lambda_V{}^2}{6}}$	0	$0<\lambda_V^2\leq 6$
C_0	0	1	0	$\lambda_V=0$
D	$-\frac{\sqrt{6}}{2\lambda_V}$	$\frac{\sqrt{6}}{2\lambda_V}$	0	$\lambda_V^2\geq 3$

Cr. P.	Ω_{DE}	w_{DE}	q	c_S^2	Q_S
C	1	$-1+\frac{\lambda_V^2}{3}$	$-1+\frac{\lambda_V^2}{2}$	1	$\frac{\lambda_V^2}{2}$
C_0	1	-1	-1	1	0
D	$\frac{3}{\lambda_V^2}$	0	$\frac{1}{2}$	1	$\frac{9}{2\lambda_V^2}$

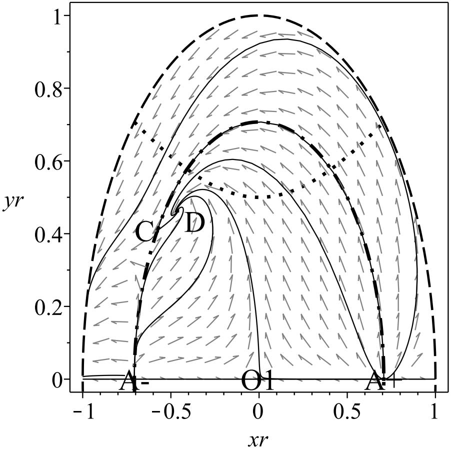
Stability conditions

Cr. P.	Exist for	Stability
C	$0 < \lambda_V^2 \leq 6$	stable node for $-\sqrt{3} < \lambda_V < 0, \lambda_g < -\lambda_V$ stable node for $0 < \lambda_V < \sqrt{3}, \lambda_g > -\lambda_V$ saddle point otherwise
C_0	$\lambda_V = 0$	stable node
D	$\lambda_V^2 \geq 3$	stable node for $-\sqrt{\frac{24}{7}} \leq \lambda_V < -\sqrt{3}, \lambda_g < -\lambda_V$ stable node for $\sqrt{3} < \lambda_V < \sqrt{\frac{24}{7}}, \lambda_g > -\lambda_V$ stable spiral for $\lambda_V < -\sqrt{\frac{24}{7}}, \lambda_g < -\lambda_V$ stable spiral for $\lambda_V > \sqrt{\frac{24}{7}}, \lambda_g > -\lambda_V$

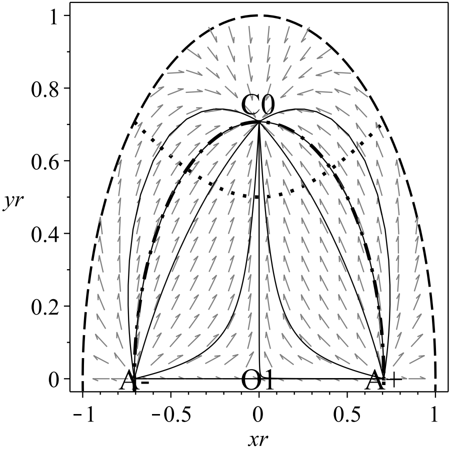
Trajectories in the y_r-x_r plane of the Poincaré phase space for the Scenario 1, that is for exponential potential and exponential coupling function. We use $\lambda_V = 1$ and λ_g arbitrary (for the numerics we choose $\lambda_g = 1$ but different λ_g's correspond to the same projection on y_r-x_r plane). In this projection the dark-energy dominated, accelerating, quintessence-like solution C is a stable solution, O_1 is saddle point, and $A^{\pm}$ are unstable.



Trajectories in the y_r-x_r plane of the Poincaré phase space for the Scenario 1, that is for exponential potential and exponential coupling function. We use $\lambda_V = 2$ and λ_g arbitrary (for the numerics we choose $\lambda_g = 1$ but different λ_g's correspond to the same projection on y_r-x_r plane). In this projection the non-accelerating, dust-like ($w_{DE} = 0$) solution D is a stable spiral, C and O_1 are saddle points, and $A^{\pm}$ are unstable.



Trajectories in the y_r-x_r plane of the Poincaré phase space for the Scenario 1, that is for exponential potential and exponential coupling function, for the specific case $\lambda_V = 0$ and λ_g arbitrary (for the numerics we choose $\lambda_g = 1$ but different λ_g's correspond to the same projection on y_r-x_r plane). In this projection the de Sitter solution C_0 is a stable node, O_1 is saddle point, and $A^{\pm}$ are unstable.



Physical discussion for Scenario 1: Exponential potential and exponential coupling function.

- Points $A^{\pm}$ exist always, that is for every values of the scenario parameters λ_V,λ_g,V_0,g_0, they are unstable or saddle, and thus they cannot be the late-time state of the universe. They correspond to a non-accelerating, dark-energy dominated universe, with a stiff dark-energy equation-of-state parameter equal to 1. Finally, the instability-related quantities c_S and Q_S do satisfy the corresponding conditions (33),(34), namely $c_S \geq 0$ and $Q_S > 0$, and thus these solutions are free of instabilities. Both of them exist in standard quintessence [Copeland, Liddle & Wands, 1998].
- Point O_1 is a saddle one and thus it cannot attract the universe at late times. It corresponds to a non-accelerating, dark-matter dominated universe, with zero total equation-of-state parameter. The instability-related quantities c_S and Q_S satisfy (33),(34), and thus this solution is free of instabilities. This point exists in standard quintessence too [Copeland, Liddle & Wands, 1998].

- Point C exists for $0 < \lambda_V^2 < 6$ and it is a stable one in the region of the parameter space given by $0 < \lambda_V < \sqrt{3}, \lambda_g > -\lambda_V$ or $-\sqrt{3} < \lambda_V < 0, \lambda_g < -\lambda_V$, and thus it can attract the universe at late times. It corresponds to a dark-energy dominated universe, with a dark-energy equation-of-state parameter lying in the quintessence regime, which can be accelerating or not according to the λ_V-value. Additionally, this solution is free of instabilities. This point exists in standard quintessence [Copeland, Liddle & Wands, 1998]. It is quite important, since it is both stable and possesses w_{DE} and q compatible with observations.
- Furthermore, we mention that in the specific case where $\lambda_V = 0$, that is in the case of constant or zero usual potential, there exist the critical point C_0, and it is always stable. It corresponds to the de Sitter solution, where the universe is accelerating and dark-energy dominated, with the dark-energy behaving like a cosmological constant ($w_{DE} = -1$), and it is free of instabilities. This point exists in standard quintessence [Copeland, Liddle & Wands, 1998]. It is quite important, since it is both stable and possesses w_{DE} and q compatible with observations.

- Since in many Galileon works the authors do not consider a potential, point C_0 just gives the late-time state of the universe in these cases [Felice & Tsujikawa, 2012, Appleby & Linder, 2012].
- Point D exists for $\lambda_V^2 \geq 3$ and in this case it is always stable, that is it can be the late-time state of the universe, and it is free of instabilities. It has the advantage that the dark-energy density parameter is in the interval $0 < \Omega_{DE} < 1$, that is it can alleviate the coincidence problem since dark energy and dark matter density parameters can be of the same order. However, it has the disadvantage that it is not accelerating and $w_{DE} = 0$, which are not favored by observations. This point exists in standard quintessence [Copeland, Liddle & Wands, 1998] too.

- Apart from the above points that exist also in standard quintessence, the scenario at hand possesses two additional critical points, namely $B^{\pm}$. They correspond to dark-energy domination, with a dark-energy equation-of-state parameter lying in the quintessence regime, where the universe is non-accelerating ($q > 0$), and they are free of instabilities. However, these points are not stable and thus they cannot attract the universe at late times.
- Finally, the present scenario possesses two critical point at infinity, namely $K^{\pm}$. They correspond to a dark-matter dominated, non-accelerating universe, with arbitrary w_{DE} but with a zero total equation-of-state parameter w_{tot}, which are also free of instabilities. They are always unstable and therefore they cannot be the late-time state of the universe.

Results for scenario 1: exponential potential and exponential coupling function.

- Galileons (simple or generalized) may survive at late-time cosmology or may be completely disappeared by the dynamics, depending on the model parameters:

  - From the above analysis we observe that at the stable critical points, C and D, we have $\dot{\phi} \to 0$, $\phi \to -\text{sign}(\lambda_V)\infty$ and thus for $\lambda_V \lambda_g > 0$ we obtain $g(\phi) \to 0$ while for $\lambda_V \lambda_g < 0$ we obtain $g(\phi) \to \infty$ (for $\lambda_V = 0$ $g(\phi)$ can be zero, finite, or infinity).
  - Similarly, for C_0 we see that for $\lambda_g > 0$ we obtain $g(\phi) \to 0$ while for $\lambda_g < 0$ we obtain $g(\phi) \to \infty$.
  - In all cases, if $\lambda_g = 0$ then obviously $g(\phi) = const$.

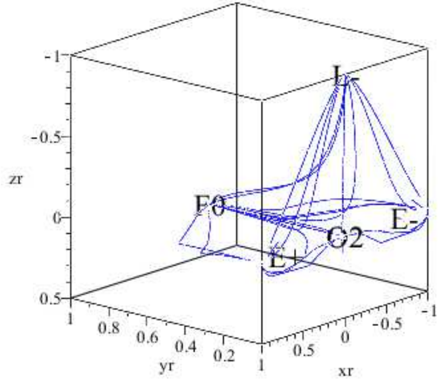
- Galileon cosmology possesses the same stable late-time solutions with standard quintessence [Copeland, Liddle & Wands, 1998].

- The corresponding observables of these solutions do not depend on the Galileon terms, but only on the usual terms and especially on the standard scalar potential (even the instability-related quantities do not depend on the Galileon terms either). This is a main result of the present work.

Late-time attractors for Scenario 2: Exponential potential and powerlaw coupling function.

Cr. P.	x_c	y_c	z_c	v_c	Exist for	Stability
F	$-\frac{\lambda_V}{6}$	$\sqrt{1-\frac{\lambda_V^2}{6}}$	0	0	$0<\lambda_V^2\leq 6$	stable node for $\lambda_V^2<3$ saddle point for $3<\lambda_V^2<6$
F_0	0	1	0	0	$\lambda_V=0$	stable (not asymptotically for $n\neq 0$) stable (asymptotically for $n=0$)
G	$-\frac{\sqrt{6}}{2\lambda_V}$	$\frac{\sqrt{6}}{2\lambda_V}$	0	0	$\lambda_V^2>3$	stable node for $3<\lambda_V^2<\frac{24}{7}$ stable spiral for $\frac{24}{7}<\lambda_V^2$

Projection of orbits on the x_r-y_r-z_r space of the Poincaré phase space for the Scenario 2, that is for exponential potential and power-law coupling function, for the specific case $\lambda_V = 0$ and n arbitrary (for the numerics we choose $n = 1$ but different n's correspond to the same projection). In this projection the de Sitter solution F_0 is the attractor, whereas L^- and $E^\pm$ are unstable and O_2 is a saddle point.



Physical interpretation for scenario 2: Exponential potential and power-law coupling function

- $E^{\pm}$ exist for every value of the parameters λ_V, n, V_0, g_0, they are saddle points, and thus they cannot be the late-time state of the universe. They correspond to a non-accelerating, dark-energy dominated universe, with a stiff dark-energy equation-of-state parameter equal to 1, and they are free of instabilities. Both of them exist in standard quintessence [Copeland, Liddle & Wands, 1998].
- Point O_2 is a saddle one and thus it cannot attract the universe at late times. It corresponds to a non-accelerating, dark-matter dominated universe, with zero total equation-of-state parameter, and it is free of instabilities. This point exists in standard quintessence too [Copeland, Liddle & Wands, 1998].

- Point F exists for $0 < \lambda_V^2 < 6$ and it is a stable one for $\lambda_V^2 < 3$ and thus it can be the late-time state of the universe. It corresponds to a dark-energy dominated universe, with a dark-energy equation-of-state parameter in the quintessence regime, which can be accelerating or not according to the λ_V-value. Additionally, this solution is free of instabilities. This point is quite important, since it is both stable and possesses w_{DE} and q compatible with observations. It exists in standard quintessence [Copeland, Liddle & Wands, 1998] too.
- F_0, which is obtained only in the case where $\lambda_V = 0$, that is in the case of constant or zero usual potential, and it is always stable. It corresponds to the de Sitter solution, where the universe is accelerating and dark-energy dominated, with the dark-energy behaving like a cosmological constant ($w_{DE} = -1$), and it is free of instabilities. Furthermore, since in many Galileon works the standard potential is not considered, point F_0 is straightforwardly the corresponding late-time state of the universe in these cases.

- Point G exists for $\lambda_V^2 \geq 3$ and in this case it is always stable, that is it can attract the universe at late times, and it is free of instabilities. It has the advantage that the dark-energy density parameter lies in the interval $0 < \Omega_{DE} < 1$, that is it can alleviate the coincidence problem, but it has the disadvantage that it is not accelerating and possesses $w_{DE} = 0$, which are not favored by observations. This point exists in standard quintessence [Copeland, Liddle & Wands, 1998] too.
- Finally, the scenario at hand possesses the critical point at infinity $L^{\pm}$ and $M^{\pm}$. They correspond to a dark-matter dominated, non-accelerating universe, with a zero total equation-of-state parameter w_{tot}, which are also free of instabilities. $M^{\pm}$ are saddle points whereas $L^{\pm}$ are unstable, and thus they cannot be the late-time state of the universe.

Results for scenario 2: exponential potential and powerlaw- coupling function.

- Galileons (simple or generalized) may survive at late-time cosmology or may be completely disappeared by the dynamics, depending on the model parameters:
  - at the stable critical points, F and G, we have $\dot{\phi} \to 0$, $\phi \to -\text{sign}(\lambda_V)\infty$ and thus for $n < 0$ we obtain $g(\phi) \to 0$ while for $n > 0$ we obtain $g(\phi) \to \infty$.
  - Similarly, for F_0 we see that for $n < 0$ we obtain $g(\phi) \to 0$ while for $n > 0$ we obtain $g(\phi) \to \infty$.
  - In all cases, if $n = 0$ then obviously $g(\phi) = const$.
- Similarly to the previous subsection, firstly we observe that this scenario possesses the same stable late-time solutions with standard quintessence [Copeland, Liddle & Wands, 1998].
- The corresponding observables of these solutions do not depend on the Galileon terms, but only on the usual terms and especially on the standard scalar potential.
- Even the instability-related quantities do not depend on the Galileon terms either. These are the main results of the present work.

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