Scalar fields and Lifshitz spacetimes

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Motivations

- Non-relativistic extensions of AdS/CFT.
- The gravitational background must have as isometries the desire symmetry algebra at the boundary.
- As far as we know, none of the theories in which these (Lifsthiz) solutions have been found, satisfy a Birkhoff’s theorem! (until know)

- What can we learn from QNM's in the AdS/CFT context?

  Horowitz-Hubeny (1999).

- Exact result possible to obtain in 2+1 BTZ,

  Birmingham-Sachs-Solodukhin (2001).

- Can some of these ideas extended to the asymptotically Lifshitz BH's?

On Lifsthiz background:

- The problem of solving the wave equation on the Lifshitz background

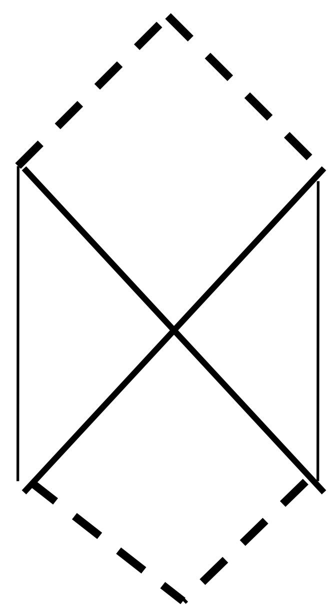
$$ds^2 = -\frac{r^{2z}}{l^{2z}}dt^2 + \frac{l^2}{r^2}dr^2 + r^2 \; d\vec{x}^2_{D-2}$$

generically reduces to solve a linear ode with more than three singular regular points.

- In particular for z=2 the solution has only three singular regular points, therefore it can be written as a hypergeometric equation.

z=2 black hole

$$ds_D^2 = -\frac{r^4}{l^4}\left(1 - \frac{r_+^2}{r^2}\right)dt^2 + \frac{l^2 dr^2}{r^2\left(1 - \frac{r_+^2}{r^2}\right)} + r^2 d\vec{x}_{D-2}^2$$



- D=4: Balasubramanian-McGreevy (2009).

Where can we find a z=2 Lifshitz BH's?

$$I = \int d^D x \sqrt{-g}\left[\sigma R - 2\Lambda + \alpha R^2 + \beta R^{\mu}_{\nu} R^{\nu}_{\mu} + c_1 R^{\mu}_{\nu} R^{\nu}_{\rho} R^{\rho}_{\mu} + c_2 R R^{\mu}_{\nu} R^{\nu}_{\mu} + c_3 R^3\right]$$

$$\Lambda = -\frac{\sigma l^{-2}}{3C_D} D(3D^6 - 26D^5 + 107D^4 - 20D^3 - 80D^2 + 368D + 208),$$

$$\alpha = \frac{\sigma l^2}{C_D}(3D^4 - 15D^3 + 6D^2 + 104D + 32),$$

$$\beta = -\frac{\sigma l^2}{C_D\,(D+2)}(D-1)(D^5 - D^4 - 20D^3 + 108D^2 + 208D + 64),$$

$$c_1 = -\frac{\sigma l^4}{3C_D}(3D+2)(D-8)(D-1)^2,$$

$$c_2 = \frac{2\sigma l^4}{C_D}\left(D^3 - 13D^2 + 8D + 4\right),$$

$$c_3 = \frac{8\sigma l^4}{3C_D}\left(4D+1\right),$$

$$C_D := D^6 - 8D^5 + 53D^4 - 130D^3 + 124D^2 + 296D + 64$$

QNM's of the z=2 Lifshitz black hole in arbitrary dimensions

$$\left(\Box - m^2\right)\Phi = \frac{1}{\sqrt{g}}(\partial_\mu \sqrt{g} g^{\mu\nu} \partial_\nu)\Phi - m^2\Phi = 0$$

$$\Phi = e^{-i\omega t} R\left(r\right) e^{i\vec{k}\cdot\vec{x}}$$

$$x = \frac{r^2 - r_+^2}{r^2} \ ,$$

$$R\left(x\right) = x^{\alpha}\left(1-x\right)^{\beta}\left[A_1 F\left(a,b,c,x\right) + A_2 x^{1-c} F\left(b-c+1, a-c+1, 2-c, x\right)\right]$$

Close to the horizon at x=0

$$\Phi \underset{x \to 0}{\sim} A_1 e^{-i\omega t} x^{-ig\omega} + A_2 e^{-i\omega t} x^{ig\omega} \underset{x \to 0}{\sim} A_1 e^{-i\omega(t+g\ln x)} + A_2 e^{-i\omega(t-g\ln x)}$$

- After asking for ingoing boundary condition at the horizon one are left with:

$$R(x) = A_1 x^{\alpha} (1-x)^{\beta} F(a,b,c,x)$$

Going to infinity

$$\Phi \underset{x\to 1}{\sim} \xi_1 \left(1-x\right)^{\beta} + \xi_2 \ \left(1-x\right)^{\beta+c-a-b} \underset{r\to +\infty}{\sim} \xi_1 r^{-\Delta_+} + \xi_2 r^{-\Delta_-}$$

$$\Delta_\pm := \frac{D \pm \sqrt{D^2 + 4m^2l^2}}{2} \ .$$

$$\xi_1 := \frac{\Gamma\left(c\right)\Gamma\left(c-a-b\right)}{\Gamma\left(c-a\right)\Gamma\left(c-b\right)} \quad \text{and} \quad \xi_2 := \frac{\Gamma\left(c\right)\Gamma\left(a+b-c\right)}{\Gamma\left(a\right)\Gamma\left(b\right)} \ .$$

Vanishing field at infinity:

$$\xi_2 = \frac{\Gamma\left(c\right)\Gamma\left(a+b-c\right)}{\Gamma\left(a\right)\Gamma\left(b\right)} = 0 \ .$$

And the frequencies are:

$$\omega = -\frac{i}{l^3}\frac{l^2k^2 + r_+^2\left(D + m^2l^2 + 4n\,(n+1) + (2n+1)\sqrt{D^2+4m^2l^2}\right)}{\sqrt{D^2+4m^2l^2} + 2\,(2n+1)}$$

- Purely imaginary and the imaginary part is always negative.

- No ringing, just damping!

Similar behavior shown in z=3 Lifshitz B.H.

Cuadros-Melgar&de Oliveira&Pellicer 2011.

- Can we extend some of these results for arbitrary values of z?

Towards Im(ω)<0 for any z

$$ds_D^2 = -\frac{r^{2z}}{l^{2z}}F(r)dt^2 + \frac{l^2 dr^2}{r^2 F(r)} + r^2 d\Sigma_{D-2}^2 \ ,$$

by a change of coordinate:

$$ds_D^2 = -r^{2z}F\left(r\right)dv^2 + 2r^{z-1}dvdr + r^2 d\Sigma_{D-2}^2 \ ,$$

$$\Phi = e^{-i\omega v}R\left(r\right)Y\left(\Sigma\right)$$

The equation for the radial dependence
Adapting the argument in Horowitz-Hubeny 1999

$$R(r) = r^{\frac{2-D}{2}} \psi(r)$$

$$\frac{d}{dr}\left(\frac{f}{r^{z-1}}\frac{d\psi}{dr}\right) - 2i\omega\frac{d\psi}{dr} - V\psi = 0 \ ,$$

where $f(r) = r^{2z-2}\left(r^2 - 1\right)$, with $r_+ = 1$

$$V := \frac{1}{4r^{z+1}}\left((D-2)(D-6+2z)\, r^{2z-2}\left(r^2-1\right) + 4\left((D-2)+m^2\right) r^{2z} + 4k^2 r^{2z-2}\right)$$

$$\int_{r=1}^{\infty} dr \left(\frac{f}{r^{z-1}}\left|\frac{d\psi}{dr}\right|^2 + V|\psi|^2\right) = -\frac{|\omega|^2\,|\psi(r=1)|^2}{Im(\omega)}$$

Constructing the Lifshitz soliton

- Double Wick rotation have been around since a long time. AdS soliton Horowitz-Myers 1998:

$$ds^2 = -\left(\frac{r^2}{l^2} - \frac{\mu}{r}\right) dt^2 + \frac{dr^2}{\frac{r^2}{l^2} - \frac{\mu}{r}} + r^2\left(dx^2 + dy^2\right)$$

Double Wick Rotation

$$ds^2 = -r^2 dx^2 + \frac{dr^2}{\frac{r^2}{l^2} - \frac{\mu}{r}} + \left(\frac{r^2}{l^2} - \frac{\mu}{r}\right) dt^2 + r^2 dy^2$$

$r > r_+$

From BTZ to AdS

$$ds^2 = -\left(r^2 - 1\right) dt^2 + \frac{dr^2}{r^2 - 1} + r^2 dx^2$$

$$ds^2 = -\left(r^2 + 1\right) dx^2 + \frac{dr^2}{r^2 + 1} + r^2 dt^2$$

$$\omega_{btz} = \pm k_{btz} - i\left(1 + 2n + \sqrt{1 + m^2}\right)$$

$$\omega_{sol} = \pm\left(1 + 2n + k_{sol} + \sqrt{1 + m^2}\right)$$

Double Wick Rotation

$$\begin{aligned} \omega_{btz} &= ik_{sol} \\ k_{btz} &= i\omega_{sol} \end{aligned}$$

From Lifshitz bh to Lifshitz Soliton (the metrics)

$$ds_D^2 = -\frac{r^4}{l^4}\left(1 - \frac{r_+^2}{r^2}\right)dt^2 + \frac{l^2 dr^2}{\left(r^2 - r_+^2\right)} + r^2 d\vec{x}_{D-2}^2 \ ,$$

$$x \to \frac{il}{r_+}\tilde{t} \text{ and } t \to \frac{il^3}{r_+^2}X \ . \quad r = r_+ \cosh\rho$$

$$ds^2 = l^2\left[-\cosh^2\rho \ d\tilde{t}^2 + d\rho^2 + \cosh^2\rho \ \sinh^2\rho \ dX^2\right] \ ,$$

From Lifshitz bh to Lifshitz Soliton (the spectrum of the scalar)

$$\omega = -\frac{i}{l^3}\frac{l^2k^2 + r_+^2\left(D + m^2l^2 + 4n\left(n+1\right) + \left(2n+1\right)\sqrt{D^2+4m^2l^2}\right)}{\sqrt{D^2+4m^2l^2} + 2\left(2n+1\right)}\ ,$$

$$k \to \frac{r_+}{l} i\omega_{sol} \quad \text{and} \quad \omega \to \frac{r_+}{l^3} ik_{sol}\ ,$$

$$\omega_{sol} = \pm\left(\left(2n + |k_{sol}| + 1\right)\sqrt{9+4m^2l^2} + 3 + m^2l^2 + 4n\left(n + |k_{sol}| + 1\right) + 2|k_{sol}|\right)^{1/2}$$

- For vanishing field at infinity, the imaginary part of the quasinormal frequency has to be negative for arbitrary z (in the family of black holes considered here). We conclude the stability of the perturbation.

- z=2 Lifshitz black holes can be found in cubic gravity theories in any dimension.

- These theories also admit black holes with other values of z. Ex:z=5 d=4

- Boundary condition at the origin? (posible to get the correct spectrum. Gonzalez-Saavedra-Vazquez 2012)

Conclusions & comments

- Im(ω)<0 Ifor bhs and Im(ω)=0 for solitons.

- The inclusion of higher curvature terms allow to find simple asymptotically Lifshitz black holes.