General Relativity with small cosmological constant from spontaneous compactification of Lovelock theory in vacuum

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Outline

- Short review of Kaluza Klein compactification
- Review of Lovelock theories
- How to obtain 4-D GR from spontaneous compactification of Lovelock theory in vacuum
- Possibility of having small cosmological constant AND small enough extra dimensions
- Shift in the effective Newton constant
- A dynamical mechanism for compactification
- Some comments about the YM coupling

Kaluza Klein theory

- Idea of unifying fundamental interactions through the dimensional reduction of General relativity.
- In order to have non-abelian gauge fields one needs extra dimensions of positive constant curvature
- For a realistic scenario one needs a small cosmological constant in 4-D and small enough extra dimensions but Einstein eq. In higher dimensions

$$R_{\mu\nu} = \frac{2\Lambda}{d-2} g_{\mu\nu}$$

- imply $\Lambda_4 = \frac{2}{d-2}\Lambda$ $\Lambda_{d-4} = \frac{d-6}{d-2}\Lambda$

- In order to avoid this clash one has to introduce matter fields, which perverts the original idea of obtaining bosonic fields from pure geometry (arbitrarines may be removed by some fundamental principle like supersymmetry)
- Is it possible to go back to the original dream of obtaining bosonic matter from pure geometry?
- A way can be to consider higher dimensional Lovelock action instead of EH action

Lovelock theory

- In 4-D the EH action is the only action built of curvature invariants leading to 2. order diff. Equations. In D>4 there are more options.
- In 5-D one can add a Gauss-Bonnet term

$$R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\alpha\beta\mu\nu}R_{\alpha\beta\mu\nu}$$

- In 7-D

$$R^3 + 3RR^{\mu\nu\alpha\beta}R_{\alpha\beta\mu\nu} - 12RR^{\mu\nu}R_{\mu\nu}$$
$$+24R^{\mu\nu\alpha\beta}R_{\alpha\mu}R_{\beta\nu} + 16R^{\mu\nu}R_{\nu\alpha}R^{\alpha}_{\mu} + 24R^{\mu\nu\alpha\beta}R_{\alpha\beta\nu\rho}R^{\rho}_{\mu}$$
$$+8R^{\mu\nu}{}_{\alpha\rho}R^{\alpha\beta}{}_{\nu\sigma}R^{\rho\sigma}{}_{\mu\beta} + 2R_{\alpha\beta\rho\sigma}R^{\mu\nu\alpha\beta}R^{\rho\sigma}{}_{\mu\nu}$$

- Lovelock Theories admit more than one maximally symmetric vacuum
- Special case is when all vacua degenerate then in odd dimensions the theory is equivalent to a Chern-Simons theory.
- Equations of motion of Lovelock theory do not imply vanishing of torsion.

Compactification in Lovelock theory in 7-D

- Action is

$$I_7 = \int_{M_7} \epsilon_{ABCDEFG} \left(c_3 R^{AB} R^{CD} R^{EF} + \frac{c_2}{3} R^{AB} R^{CD} e^E e^F + \frac{c_1}{5} R^{AB} e^C e^D e^E e^F + \frac{c_0}{7} e^A e^B e^C e^D e^E e^F \right) e^G ,$$

- With the ansatz $M_7 = M_4 \times K_3$ the projection on M_4 of e.o.m are

$$\frac{1}{8\pi \tilde{G}_4} \left(G_{\mu\nu} + \Lambda_4 g_{\mu\nu} \right) = 0$$

$$\tilde{\kappa}_4 := (16\pi \tilde{G}_4)^{-1} = 4!\, Vol(K_3)(c_1 + c_2 \Lambda_3)$$

$$\Lambda_4 = -3 \frac{5c_0 + c_1 \Lambda_3}{c_1 + c_2 \Lambda_3}.$$

- The projection on the compact manifold gives

$$(c_2 + 3c_3\Lambda_3)\left(R^{\mu\nu\lambda\rho}R_{\mu\nu\lambda\rho} - 4R^{\mu\nu}R_{\mu\nu} + R^2\right) + 24\left[\frac{c_0(5\Lambda_3 c_2 - 15c_1) - \Lambda_3 c_1(5c_1 - \Lambda_3 c_2)}{c_1 + c_2\Lambda_3}\right] = 0\ .$$

- When the cubic term is switched on there is the possibility to eliminate the constraint and to reduce to pure GR (minimal dimension is D=7) i.e.

$$\Lambda_3 = -c_2/(3c_3),$$

$$\tilde{c}_0 = \frac{\tilde{c}_1\tilde{c}_2(15\tilde{c}_1 - \tilde{c}_2^2)}{15(9\tilde{c}_1 + \tilde{c}_2^2)}$$

$$\tilde{c}_i = c_i/c_3$$

- This class of theories admit spontaneous compactification reducing to pure GR without correction or constraints

- The effect of the cuadratic and cubic term just amout to a redefinition of the effective Newton constant ant of the cosmological constants given by

$$\tilde{\kappa}_4 = \frac{1}{16\pi\tilde{G}_4} = 4!\,Vol(K_3)c_3(\tilde{c}_1 - \frac{\tilde{c}_2^2}{3})$$

$$\Lambda_4 = -\frac{6\tilde{c}_1\tilde{c}_2}{9\tilde{c}_1 + \tilde{c}_2^2}\,.$$

$$\Lambda_3 = -\tilde{c}_2/3\,.$$

- One can see that now it is possible to have small 4-D cosmological constant and smal extra dimensions without introducing any matter fields. This is not possible in standard KK reduction.

The condition $|\Lambda_4/\Lambda_3| \ll 1$ amounts to requiring $|9\tilde{c}_1/\tilde{c}_2^2| \ll 1$

- Due to the shift the 4-D Newton constant can be positive even if c_1 is negative or absent

Maximally symmetric spacetimes versus compactification

- Why is do we live in a compactified universe instead of a nice maximally symmtric one?
- In standard KK theory it is not easy to explain why the maximally symmetric spacetime if preferred instead of the compactified one
- In our case the theory has up to 3 maximally symmetric vacua.

$$R^{AB} = \lambda_7 e^A e^B$$

$$P(\lambda_7) := \lambda_7^3 + \tilde{c}_2\lambda_7^2 + \tilde{c}_1\lambda_7 + \tilde{c}_0 = 0$$

- Generally the 7-D Newton constant is not given by c_1 indeed ilearizing the field eq. Around a max. Symm. vacuum

$$c_3 P'(\bar{\lambda}_7)\ \epsilon_{A_1 \ldots A_6 B}\ \delta\left(R^{A_1 A_2} - \bar{\lambda}_7 e^{A_1} e^{A_2}\right) \bar{e}^{A_3} \ldots \bar{e}^{A_6} = 0\ .$$

$$\bar{\kappa}_7 := \frac{1}{16\pi \bar{G}_7} = 4! c_3 P'(\bar{\lambda}_7)$$

- So the graviton in 7-D could behave like a graviton or a ghost depending on the slope of the polynomial
- In the cae that the 7-D graviton is ghost the max. Symm. vacuum may decay to a compactified spacetime through ghost condensation
- Also if the graviton is a particle the max. symm. Spacetime may be metasatble
- One can further restrict the arbitrariness of the free parameters by requiring $|\Lambda_4| \ll |\Lambda_3|$, $9\,|\tilde{c}_1| \ll \tilde{c}_2^2$. and a well defined 4-D graviton $c_3 < 0$ requiring also $\Lambda_3 > 0$ (i.e. $\tilde{c}_2 < 0$)
- There different scenarios according to the sign of the 4-D cosmological constant

- 1)Negative 4-D cosmological constant: the polynomial $P(\lambda_7)$ has only one real positive root and the 7-D graviton is a ghost
- 2) vanishing 4-D cosmological constant: there is one simple root positive and a doubly degenerate vanishing one. The graviton at the simple root behaves like a ghost. For the degenerate root the propagator is ill defined
- 3) Positive 4-D cosmological constant: There are 3 real roots , $\lambda_7^{(1)} < 0 < \lambda_7^{(2)} < \lambda_7^{(3)}$ for the smallest and largest root
- the graviton behaves like a ghost and for $\lambda_7^{(2)}$ it behaves like a particle but this vacuum turns out to be metastable and tunnels to . $dS_4 \times S^3$.

Tunneling

- One can estimate to more probable configuration S^7 or $S^4 \times S^3$, by evaluating the euclidean action

In the case of $S^4 \times S^3$, for theories with $x := 9\tilde{c}_1\tilde{c}_2^{-2} = (2\Lambda_3/\Lambda_4 - 1)^{-1} \ll 1$ (i.e., $\Lambda_4 \ll \Lambda_3$), the Euclidean action, up to an overall positive factor, becomes

$$I_{4,3} \simeq -c_3 x^{-1} \; ;$$

while for S^7, of curvature given by $\lambda_7^{(2)} = (\tilde{c}_1/15)^{1/2} + \mathcal{O}(x)$, the action is given by

$$I_7 \simeq -c_3 x^{-3/4} \; .$$

So the compactified spacetime is more probable

- Notice that this mechanism does not occur for GR

$$I_{EH}(M_7) \; = \; (20\pi G_7)^{-1}\Lambda_7 Vol(M_7)$$

$$I_{EH}(S^7)/I_{EH}(S^4 \times S^3) = 3^{3/2}/4 > 1.$$

- In sum requiring compatibility with realistic 4-D scenario the compactifed vacuum turn out to be prefered to the maximally symmetric one since it would decay either by ghost condensation or tunneling

Gauge coupling

- Since the gravitational theory in higher dimensions is generally covariant and of second order in the derivatives of the metric the massless gauge fields are gauranteed to be gauge invariant and at linearized level fulfill the YM equations with

$$4(3c_1 + c_2\Lambda_3)\partial^\mu F^{(I)}_{\mu\nu} = 0 \ ,$$

$$F^{(I)}_{\mu\nu} = \partial_{[\mu} A^{(I)}_{\nu]} .$$