Delta Gravity: A Finite Quantum Gravity Field Theory Model

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Motivations

Delta Gravity, in Cosmology, shows accelerated expansion of the Universe without a cosmological constant, which in turns means that we do not need to consider the problem of dark energy.

Delta Gravity offers a possible solution to the long sought Quantum Theory of Gravity.

Delta Gravity preserves all well tested classical results of Einstein's General Theory of Relativity in vacuum.

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Delta Transformation

In this work we will study a modification of models that consist in the application of a variation that we will define as $\tilde{\delta}$. The particular point with this variation is that when applied to a field (function, tensor, etc.) it will give a new element that we define as $\tilde{\delta}$ fields which is an entire new independent object from the original:

$$\tilde{\delta}(\Phi) = \tilde{\Phi} \tag{1}$$

and to indicate this, is that we call this variation 'delta tilde' $\tilde{\delta}$.
So, when we consider general coordinate transformations or diffeomorphism in its infinitesimal form we have:

$$\begin{aligned} x'^{\mu} &= x^{\mu} - \xi_0^{\mu}(x) \\ \delta x^{\mu} &= -\xi_0^{\mu}(x) \end{aligned} \tag{2}$$

Where δ is the general coordinate transformation. Now, we define:

$$\xi_1^{\mu}(x) \equiv \tilde{\delta}\xi_0^{\mu}(x) \tag{3}$$

Delta Transformation: Gauge Transformation

We have a theory with two fields, the first is just the usual gravitational field $g_{\mu\nu}(x)$ and a second one $\tilde{g}_{\mu\nu}(x)$ which corresponds to the $\tilde{\delta}$ variation of the first. We will have two transformations.

$$\delta g_{\mu\nu}(x) = \xi_{0\mu;\nu} + \xi_{0\nu;\mu} \quad (4)$$

$$\delta \tilde{g}_{\mu\nu}(x) = \xi_{1\mu;\nu} + \xi_{1\nu;\mu} + \tilde{g}_{\mu\rho}\xi^{\rho}_{0,\nu} + \tilde{g}_{\nu\rho}\xi^{\rho}_{0,\mu} + \tilde{g}_{\mu\nu,\rho}\xi^{\rho}_{0} \quad (5)$$

where $\xi^{\mu}_{0}(x)$ and $\xi^{\mu}_{1}(x)$ are infinitesimal contravariant vectors of the gauge transformations. It has been verified that they form a closed algebra.

Classical Modified Model: The New Action

We start by considering a model which is based on a given action $S_0[\phi_I]$ where ϕ_I are generic fields, then we add to it a piece which is equal to an $\tilde{\delta}$ variation with respect to the fields and we let $\tilde{\delta}\phi_J = \tilde{\phi}_J$ so that we have:

$$S[\phi,\tilde{\phi}] = S_0[\phi] + \kappa_2 \int d^4x \frac{\delta S_0}{\delta\phi_I(x)}[\phi]\tilde{\phi}_I(x) \tag{6}$$

with κ_2 an arbitrary constant.
The equations of motion are:

$$\frac{\delta S_0}{\delta\phi_I(x)}[\phi] = 0 \tag{7}$$

and,

$$\int d^4x \frac{\delta^2 S_0}{\delta\phi_I(y)\delta\phi_J(x)}[\phi]\tilde{\phi}_J(x) = 0 \tag{8}$$

Classical Modified Model: Invariance of Delta Gravity Action

In this work, we will investigate the $\tilde{\delta}$ Gravity action, obtained by the procedure sketched above:

$$S[g,\tilde{g}] = \int d^dx\sqrt{-g}\left(-\frac{1}{2\kappa}R\right) + \kappa_2\int\left(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\right)\sqrt{-g}\tilde{g}_{\mu\nu}d^dx \quad (9)$$

We can see that (9) is obviously invariant under transformations generated by ξ_0^ρ, since these are general coordinate transformations and we declared $\tilde{g}_{\mu\nu}$ to be a covariant two tensor. Under transformations generated by $\xi_1^\rho(\delta_1)$, $g_{\mu\nu}$ does not change, so we have:

$$\begin{aligned}
\delta_1 S(g,\tilde{g}) &= \kappa_2\int\left(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\right)\sqrt{-g}(\delta_1\tilde{g}_{\mu\nu})d^dx \\
&= \kappa_2\int\left(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\right)\sqrt{-g}(\xi_{1\mu;\nu} + \xi_{1\nu;\mu})d^dx \\
&= -2\kappa_2\int\left(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\right)_{;\nu}\sqrt{-g}\xi_{1\mu}d^dx = 0 \quad (10)
\end{aligned}$$

Quantum Modified Model: One Loop Model

The generating functional for this model is:

$$Z(j,\tilde{j}) = e^{iW(j,\tilde{j})} = \int \mathcal{D}\phi\mathcal{D}\tilde{\phi}e^{i\left(S_0+\int d^Nx\frac{\delta S_0}{\delta\phi_I}\tilde{\phi}_I+\int d^Nx(j_I(x)\phi_I(x)+\tilde{j}_I(x)\tilde{\phi}_I(x))\right)} \quad (11)$$

integrating the field $\tilde{\phi}$ we get:

$$Z(j,\tilde{j}) = \int \mathcal{D}\phi e^{i\left(S_0+\int d^Nxj_I(x)\phi_I(x)\right)}\delta\left(\frac{\delta S_0}{\delta\phi_I(x)}+\tilde{j}_I(x)\right) \quad (12)$$

Let φ_I solves the classical equation of motion:

$$\frac{\delta S_0}{\delta\phi_I(x)}|_{\varphi_I}+\tilde{j}_I(x) = 0 \quad (13)$$

and using:

$$\delta\left(\frac{\delta S_0}{\delta\phi_I(x)}+\tilde{j}_I(x)\right) = \overset{-1}{\det}\left(\frac{\delta^2S_0}{\delta\phi_I(x)\delta\phi_J(y)}|_{\varphi_I}\right)\delta(\phi_I-\varphi_I) \quad (14)$$

Quantum Modified Model: One Loop Model

Therefore:

$$Z(j,\tilde{j}) = \int \mathcal{D}\phi e^{i(S_0+\int d^N x j_I(x)\phi_I(x))}\delta\left(\frac{\delta S_0}{\delta\phi_I(x)} + \tilde{j}_I(x)\right) \tag{15}$$
$$= e^{i(S_0(\varphi)+\int d^N x j_I(x)\varphi_I(x))} \overset{-1}{\det}\left(\frac{\delta^2 S_0}{\delta\phi_I(x)\delta\phi_J(y)}|_{\varphi_I}\right)$$

$$W(j,\tilde{j}) = S_0(\varphi) + \int d^N x j_I(x)\varphi_I(x) + i\mathrm{Tr}\left(\log\left(\frac{\delta^2 S_0}{\delta\phi_I(x)\delta\phi_J(y)}|_{\varphi_I}\right)\right) \tag{16}$$

Define:

$$\Phi_I(x) = \frac{\delta W}{\delta j_I(x)} = \varphi_I(x)$$
$$\tilde{\Phi}_I(x) = \frac{\delta W}{\delta \tilde{j}_I(x)}$$

Quantum Modified Model: One Loop Model

The effective action is defined by:

$$\Gamma(\Phi,\tilde{\Phi}) = W(j,\tilde{j}) - \int d^N x \left\{ j_I(x)\Phi_I(x) + \tilde{j}_I(x)\tilde{\Phi}_I(x) \right\}$$

We get, using equations (13) and (16):

$$\Gamma(\Phi,\tilde{\Phi}) = S_0(\Phi) + \int d^N x \frac{\delta S_0}{\delta \Phi_I(x)} \tilde{\Phi}_I(x) + i\mathrm{Tr}\left(\log\left(\frac{\delta^2 S_0}{\delta \Phi_I(x) \delta \Phi_J(y)} \right) \right) \quad (17)$$

This is the exact effective action for $\tilde{\delta}$ theories.

Delta Gravity: Classical Equations of Motion

We start with the following lagrangian:

$$L[g_{\mu\nu}] \quad = \quad \sqrt{-g}\left[-\frac{1}{2\kappa}R + \kappa_2\left(G^{\mu\nu}\right)\tilde{g}_{\mu\nu}\right] \qquad (18)$$

with $\kappa = \frac{8\pi G}{c^4}$.
If we variate this action, we obtain the equations of motion:

$$\begin{aligned} G^{\mu\nu} &= 0 \\ F^{(\mu\nu)(\alpha\beta)\rho\lambda}D_\rho D_\lambda \tilde{g}_{\alpha\beta} &= 0 \end{aligned} \qquad (19)$$

with:

$$\begin{aligned} F^{(\mu\nu)(\alpha\beta)\rho\lambda} &= P^{((\rho\mu)(\alpha\beta))}g^{\nu\lambda} + P^{((\rho\nu)(\alpha\beta))}g^{\mu\lambda} - P^{((\mu\nu)(\alpha\beta))}g^{\rho\lambda} - P^{((\rho\lambda)(\alpha\beta))} \\ P^{((\alpha\beta)(\mu\nu))} &= \frac{1}{4}\left(g^{\alpha\mu}g^{\beta\nu} + g^{\alpha\nu}g^{\beta\mu} - g^{\alpha\beta}g^{\mu\nu}\right) \end{aligned}$$

Where $(\mu\nu)$ tells us that the μ and ν are in a totally symmetric combination.

Delta Gravity: Divergent Part Effective Action

We use the background field method:

$$\begin{aligned} g_{\mu\nu} &\rightarrow g_{\mu\nu} + h_{\mu\nu} \\ \tilde{g}_{\mu\nu} &\rightarrow \tilde{g}_{\mu\nu} + \tilde{h}_{\mu\nu} \end{aligned} \tag{21}$$

When we calculate the quadratic part in the quantum gravitational fields, $h_{\mu\nu}$ and $\tilde{h}_{\mu\nu}$, we obtain:

$$L_{quad} = \frac{1}{2}\sqrt{-g}\vec{h}^T_{(\alpha\beta)} P^{((\alpha\beta)(\mu\nu))} \left(\left[K^{(\gamma\varepsilon)}_{(\mu\nu)} \right]^{(\lambda\eta)} \nabla_\lambda \nabla_\eta + \left[W^{(\gamma\varepsilon)}_{(\mu\nu)} \right] \right) \vec{h}_{(\gamma\varepsilon)} \tag{22}$$

We calculate the divergent part of the effective action following an algorithm developed by **Petr I. Pronin and Konstantin V. Stepanyantz. Nuclear Physics B. Vol** 485 **(**1997**), page** 517-544. We get:

$$L^{div}_Q = \sqrt{-g}\frac{\hbar c}{\varepsilon}\left(\frac{1}{60}R^2 + \frac{7}{10}R_{\alpha\beta}R^{\alpha\beta} \right) \tag{23}$$

with $\varepsilon = 8\pi^2(N-4)$.

Ghosts

To study the existence of ghosts in the model we will study small perturbations to flat space. This is done by taking expression (22) and putting the backgrounds equal to the Minkowski metric $g_{\mu\nu} = \eta_{\mu\nu}$ and $\tilde{g}_{\mu\nu} = \eta_{\mu\nu}$, thus obtaining:

$$S[h,\tilde{h}] = -\frac{1}{2\kappa}\int d^4x P^{((\alpha\beta)(\mu\nu))}\left(\frac{(1-\kappa_2)}{2}\partial_\rho h_{\alpha\beta}\partial^\rho h_{\mu\nu} + \kappa_2 \partial_\rho \tilde{h}_{\alpha\beta}\partial^\rho h_{\mu\nu}\right) \quad (24)$$

The Hamiltonian associated to this action is not diagonal, so we redefine the fields in order to make it diagonal

$$\begin{aligned} h_{\mu\nu} &= A\bar{h}^1_{\mu\nu} + B\bar{h}^2_{\mu\nu} \\ \tilde{h}_{\mu\nu} &= C\bar{h}^1_{\mu\nu} + D\bar{h}^2_{\mu\nu} \end{aligned} \quad (25)$$

where A, B, C and D are real constants so that the new fields, $\bar{h}^1$ and $\bar{h}^2$, are real fields.

Ghosts

The final result is shown below. It can be demonstrated that imposing the above criteria, it is inevitable that one (and only one) of two fields will be a ghost. We make the choice of $\bar{h}^2$ as the corresponding ghost.
We get for our action:

$$S[\bar{h}^1, \bar{h}^2] = \frac{1}{2\kappa}\int d^4x P^{((\alpha\beta)(\mu\nu))}\left(\frac{1}{2}\bar{h}^1_{\alpha\beta}\partial^2\bar{h}^1_{\mu\nu} - \frac{1}{2}\bar{h}^2_{\alpha\beta}\partial^2\bar{h}^2_{\mu\nu}\right) \tag{26}$$

The Hamiltonian is:

$$H = \int \frac{d^3p}{4\kappa}E_{\mathbf{p}}(b^{1+}_{AB}b^{1AB} - b^{2+}_{AB}b^{2AB}) \tag{27}$$

The problem with these models are that when they are quantized either there is a lost of unitarity or there are negative energy which mean lost of stability. Looking at (26) we find that the propagators of $\bar{h}^1$ and $\bar{h}^2$ are respectively:

$$-2\kappa P^{-1}_{((\alpha\beta)(\mu\nu))}\frac{i}{p^2 - i\varepsilon} \tag{28}$$

$$2\kappa P^{-1}_{((\alpha\beta)(\mu\nu))}\frac{i}{p^2 \pm i\varepsilon} \tag{29}$$

where $\pm$ in the phantom propagator, $\bar{h}^2$, will decide whether unitarity and negative energy solutions or nonunitary and positive energy solutions will be present in the model.

Finite Quantum Corrections

We have two types of quantum corrections, the local; and non local terms
The non local terms are:

$$\sqrt{-g}R_{\mu\nu}\ln\left(\frac{\nabla^2}{\mu^2}\right)R^{\mu\nu}$$
$$\sqrt{-g}R\ln\left(\frac{\nabla^2}{\mu^2}\right)R \tag{30}$$

these are zero.
The local terms correspond to a series expansion in powers of the curvature, we consider the most simpler ones i.e. the cubic terms:

$$L_Q^{fin} = \sqrt{-g}(c_1\ R_{\mu\nu\lambda\sigma}R^{\alpha\beta\lambda\sigma}R^{\mu\nu}{}_{\alpha\beta}\ +\ c_2\ R^{\mu\nu}{}_{\lambda\sigma}R_{\mu\alpha}{}^{\lambda\beta}R^{\alpha\sigma}{}_{\nu\beta}\ +\ c_3\ R_{\mu\nu}R^{\mu\alpha\beta\gamma}R^{\nu}{}_{\alpha\beta\gamma} \tag{31}$$
$$+\ c_4\ RR_{\mu\nu\lambda\kappa}R^{\mu\nu\lambda\kappa}) \tag{32}$$

These type of corrections will affect the equations of motion for $\tilde{g}_{\mu\nu}$. So, using (??) we obtain:

$$F^{(\mu\nu)(\alpha\beta)\rho\lambda}D_\rho D_\lambda\tilde{g}_{\alpha\beta} = -\frac{1}{\kappa_2}\left(M^{(\mu\nu)} + c_1N^{(\mu\nu)} + c_2B^{(\mu\nu)} + 3\left\{D_\rho\ ,\ D\sigma\right\}E^{[\sigma\mu][\nu\rho]}\right) \tag{33}$$

Finite Quantum Corrections

with:

$$M^{(\mu\nu)} = \frac{1}{2}\left(D_\alpha D^\nu A^{(\alpha\mu)} + D_\alpha D^\mu A^{(\alpha\nu)} - D_\alpha D^\alpha A^{(\mu\nu)} - g^{\mu\nu} D_\alpha D_\beta A^{(\alpha\beta)}\right) \quad (34)$$

$$A^{(\mu\nu)} = c_3 R^{\mu\alpha\beta\gamma} R^\nu{}_{\alpha\beta\gamma} + c_4 g^{\mu\nu} R^{\alpha\beta\gamma\epsilon} R_{\alpha\beta\gamma\epsilon} \quad (35)$$

$$N^{(\mu\nu)} = \frac{1}{2} g^{\mu\nu} R_{\rho\epsilon\lambda\sigma} R^{\lambda\sigma\alpha\beta} R_{\alpha\beta}{}^{\rho\epsilon} + 3 R_{\rho\epsilon\lambda\sigma} R_\alpha{}^{\nu\epsilon\rho} R^{\alpha\mu\lambda\sigma} \quad (36)$$

$$B^{(\mu\nu)} = \frac{1}{2} g^{\mu\nu} R_{\rho\epsilon\lambda\sigma} R^{\rho\alpha\lambda\beta} R_\alpha{}^{\sigma\epsilon}{}_\beta + 3 R_{\rho\epsilon\lambda\sigma} R^{\nu\sigma\rho}{}_\beta R^{\mu\epsilon\beta\lambda} \quad (37)$$

$$E^{[\sigma\mu][\nu\rho]} = c_1 R^{\sigma\mu}{}_{\alpha\beta} R^{\alpha\beta\nu\rho} + \frac{1}{2} c_2 \left(R^\nu{}_\alpha{}^\sigma{}_\beta R^{\rho\beta\alpha\mu} - R^\rho{}_\alpha{}^\sigma{}_\beta R^{\nu\beta\alpha\mu}\right) \quad (38)$$

Conclusions

We have shown that the $\tilde{\delta}$ transformation, applied to any theory, produce physical models that live only at one loop.

In Delta Gravity we calculated the divergent part of the action and we obtained that it is zero in vacuum implying a finite theory

We show that this model has ghosts. These ghosts could be related to phantom fields which have been proposed by some authors as a possible explanation to describe the accelerated expansion rate of the Universe. What's more, in cosmology, Delta Gravity shows accelerated expansion of the Universe as was found in **J.Alfaro, Delta-gravity and Dark Energy, arXiv:1006.5765v2**

Conclusions

We want to point out that supergravity with matter is finite at one Loop level. According to the general argument developed in this work, $\tilde{\delta}$ supergravity will be a one Loop model which have a strong possibility to be a finite quantum model of gravity plus matter and also it may solve the instability of negative energies since in supersymmetry one has an hermitian charge whose square is equal to the Hamiltonian operator meaning that the Hamiltonian is positive definite.