Delta-gravity

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Motivation

- Delta Gauge Theories,J. A, "bv gauge theories", hep-th 9702060,J. A and P. Labraña,Phys. Rev. D 65, 045002 (2002)

  → Classical equations of motion are satisfied in the full Quantum theory

  → They live at one loop. All higher loop corrections are strictly zero.

  → They are obtained through the extension of the former symmetry of the model, introducing an extra symmetry(δ), which is formally obtained as the variation of the original symmetry.

- General Relativity without matter and without a cosmological constant is finite at one loop, G. 'tHooft and M. Veltman, "One-loop divergencies in the theory of gravitation", Annales de l'I.H.P, section A, tome 20, number 1 (1974), pag. 69-94.

- J.A., P. González and R. Avila, "A Finite Quantum Gravity Field Theory Model", gr-qc 1009.2800, Class. Quantum Grav. 28 (2011) 215020.

- J.A. Delta Gravity and Dark Energy, gr-qc 1006.5765, Phys. Lett B709(2012)101.

Definition of Delta gravity

We use the metric convention (-+++).

$\kappa = \frac{8\pi G}{c^4}$ and $\bar{\kappa}_2$ is an arbitrary constant.

$$
\begin{aligned}
S(g,\tilde{g},\lambda) = \int d^d x \sqrt{-g}(-\frac{1}{2\kappa} R + \mathcal{L}_M) + \\
\kappa_2 \int \left[\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) + \kappa T_{\mu\nu} \right] \sqrt{-g}\, \tilde{g}^{\mu\nu} d^d x \\
\kappa_2 \kappa \int \sqrt{-g}\, (\lambda^{\mu;\nu} + \lambda^{\nu;\mu}) T_{\mu\nu} d^d x
\end{aligned}
\tag{1}
$$

Symmetries

Action (1) is invariant under the following transformations(δ),

$$
\begin{aligned}
\delta g_{\mu\nu} &= g_{\mu\rho}\xi^{\rho}_{0,\nu} + g_{\nu\rho}\xi^{\rho}_{0,\mu} + g_{\mu\nu,\rho}\xi^{\rho}_{0} = \xi_{0\mu;\nu} + \xi_{0\nu;\mu} \\
\delta \tilde{g}_{\mu\nu}(x) &= \xi_{1\mu;\nu} + \xi_{1\nu;\mu} + \tilde{g}_{\mu\rho}\xi^{\rho}_{0,\nu} + \tilde{g}_{\nu\rho}\xi^{\rho}_{0,\mu} + \tilde{g}_{\mu\nu,\rho}\xi^{\rho}_{0} \\
\delta \lambda_{\mu} &= -\xi_{1\mu} + \lambda_{\rho}\,\xi^{\rho}_{0,\mu} + \lambda_{\mu,\rho}\xi_{0}
\end{aligned}
\tag{2}
$$

From now on we will fix the gauge $\lambda_\mu = 0$. This gauge preserves general coordinate transformations but fixes completely the extra symmetry with parameter $\xi_{1\mu}$.

Equations of motion

The equation of motion for $\tilde{g}^{\mu\nu}$ is:

$$S^{\gamma\sigma} + \frac{1}{2}(R\tilde{g}^{\gamma\sigma} - g_{\mu\nu}R^{\sigma\gamma}\,\tilde{g}^{\mu\nu}) - \frac{1}{2}g^{\sigma\gamma}\frac{1}{\sqrt{-g}}(\sqrt{-g}\,\nabla_\nu\,\tilde{g}^{\mu\nu})_{,\mu} + \frac{1}{4}g^{\sigma\gamma}\frac{1}{\sqrt{-g}}(\sqrt{-g}\,g^{\alpha\sigma}\,g_{\mu\nu}\nabla_\sigma\,\tilde{g}^{\mu\nu})_{,\alpha} = \kappa\frac{\delta T_{\mu\nu}}{\delta g_{\gamma\sigma}}\tilde{g}^{\mu\nu} \tag{3}$$

where

$$S^{\sigma\gamma} = (U^{\sigma\beta\gamma\rho} + U^{\gamma\beta\sigma\rho} - U^{\sigma\gamma\beta\rho})_{;\rho\beta} \qquad U^{\alpha\beta\gamma\rho} = \frac{1}{2}\left[g^{\gamma\rho}(\,\tilde{g}^{\beta\alpha} - \frac{1}{2}g^{\alpha\beta}g_{\mu\nu}\,\tilde{g}^{\mu\nu})\right] \tag{4}$$

The equation of motion for $g_{\mu\nu}$ is Einstein equation:

$$\left(R_{\mu\nu} - \frac{1}{2}\,g_{\mu\nu}R\right) + \kappa T_{\mu\nu} = 0 \tag{5}$$

Covariant derivatives as well as raising and lowering of indices are defined using $g_{\mu\nu}$. Notice that outside the sources($T_{\mu\nu} = 0$), a solution of (3) is $\tilde{g}^{\mu\nu} = \lambda g^{\mu\nu}$, for a constant λ. We will have $\tilde{g}^{\mu\nu} = g^{\mu\nu}$, assuming that both fields satisfy the same boundary conditions.

Particle motion in the gravitational field

The action of a particle in a gravitational field is

$$\frac{1}{2\kappa}\int d^n y\sqrt{-g}R - m\int dt\sqrt{-g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu}$$

The variation in x^μ produces the geodesic equation.

Instead in δ gravity we have:

$$\frac{1}{2\kappa}\int d^n y\sqrt{-g}R - m\int dt\sqrt{-g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu} +$$

$$\kappa_2\int d^n y\sqrt{-g}(G_{\mu\nu}+\kappa T_{\mu\nu})\tilde{g}^{\mu\nu}$$

where:

$$T_{\mu\nu}(y) = \frac{m}{2\sqrt{-g}}\int dt\frac{\dot{x}_\mu\dot{x}_\nu}{\sqrt{-g_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta}}\delta(y-x)$$

Notice that $T_{\mu\nu}$ is t-parametrization invariant. Call $\kappa_2' = \kappa_2\kappa$. It is dimensionless.

The new action has a term:

$$S_p = m\int \frac{dt}{\sqrt{-g_{\alpha\beta}\dot{x}^{\alpha}\dot{x}^{\beta}}}\dot{x}_{\mu}\dot{x}_{\nu}(g^{\mu\nu}+\frac{\kappa_2{}'}{2}\tilde{g}^{\mu\nu}) = m\int \frac{dt}{\sqrt{-g_{\alpha\beta}\dot{x}^{\alpha}\dot{x}^{\beta}}}\bar{\mathfrak{g}}_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu} \tag{6}$$

Since far from the sources, we must have free particles in Minkowski space, it follows that we are describing the motion of a particle of mass m':

$$m' = m(1+\frac{\kappa_2{}'}{2})$$

Since outside the sources $\tilde{g}^{\mu\nu} = g^{\mu\nu}$, the geodesic equation outside the sources is the same as Einstein's.

Equation of motion for massive particles:

$$\frac{d(\dot{x}^{\mu}\dot{x}^{\nu}\bar{\mathfrak{g}}_{\mu\nu}\dot{x}^{\beta}g_{\alpha\beta}+2\dot{x}^{\beta}\bar{\mathfrak{g}}_{\alpha\beta})}{d\tau} - \frac{1}{2}\dot{x}^{\mu}\dot{x}^{\nu}\bar{\mathfrak{g}}_{\mu\nu}\dot{x}^{\beta}\dot{x}^{\gamma}g_{\beta\gamma,\alpha} - \dot{x}^{\mu}\dot{x}^{\nu}\bar{\mathfrak{g}}_{\mu\nu,\alpha} = 0 \quad , \dot{x}^{\mu}\dot{x}^{\nu}g_{\mu\nu} = -1 \tag{7}$$

Massless particle moves in a null geodesic of $\mathfrak{g}_{\mu\nu}$=$g^{\mu\nu}+\kappa_2'\tilde{g}^{\mu\nu}$:

$$K = \dot{x}^{\mu}\dot{x}^{\nu}\mathfrak{g}_{\mu\nu} = 0 \quad , \; L_0 = -\int dt\frac{1}{4}(\dot{x}^{\mu}\dot{x}^{\nu}\mathfrak{g}_{\mu\nu})$$

Distances and time intervals

Remark 1. Eq.(7) satisfies the important property of preserving the form of the proper time in a particle in free fall. Notice that in our case the quantity that is constant using (7) is $\dot{x}^\mu \dot{x}^\nu g_{\mu\nu}$. **This single out this definition of proper time and not other.**

Proper time:

$$d\tau = \sqrt{-g_{00}}\, dx^0 \tag{8}$$

Following Landau, the change in x^0 for a roundtrip of a light ray from a point with coordinates x^α to a neighboring point with coordinates $x^\alpha + dx^\alpha$ is:

$$\Delta x^0 = \frac{2}{\mathfrak{g}_{00}} \sqrt{(\mathfrak{g}_{0\alpha}\mathfrak{g}_{0\beta} - \mathfrak{g}_{\alpha\beta}\mathfrak{g}_{00})}$$

$$\begin{aligned} dl &= \sqrt{\frac{g_{00}}{\mathfrak{g}_{00}}} \sqrt{\mathfrak{g}_{\alpha\beta} - \frac{\mathfrak{g}_{0\alpha}\mathfrak{g}_{0\beta}}{\mathfrak{g}_{00}}} \\ dl^2 &= \gamma_{ij} dx^i dx^j, \\ \gamma_{ij} &= \frac{g_{00}}{\mathfrak{g}_{00}} (\mathfrak{g}_{ij} - \frac{\mathfrak{g}_{0i}\mathfrak{g}_{0j}}{\mathfrak{g}_{00}}) \end{aligned} \tag{9}$$

Energy momentum tensor for a perfect fluid

$$T_{\mu\nu}=pg_{\mu\nu}+(p+\rho)U_\mu U_\nu,\ g^{\mu\nu}U_\mu U_\nu=-1 \tag{10}$$

Then:

$$\frac{\delta T_{\mu\nu}}{\delta g_{\gamma\sigma}}\tilde{g}^{\mu\nu}=p\tilde{g}^{\gamma\sigma}+\frac{1}{2}(p+\rho)(U^\gamma U_\nu\tilde{g}^{\sigma\nu}+U^\sigma U_\nu\tilde{g}^{\gamma\nu}) \tag{11}$$

Friedman-Robertson-Walker metric

In this case, assuming flat three dimensional metric:

$$-ds^2=dt^2-R(t)^2\{dr^2+r^2d\theta^2+r^2\sin^2\theta d\phi^2\}$$
$$-d\,\tilde{s}^2=\tilde{A}(t)dt^2-\tilde{B}(t)\{dr^2+r^2d\theta^2+r^2\sin^2\theta d\phi^2\}$$

Equations (3,11) give:

$$-\dot{R}\dot{\tilde{B}}-\frac{1}{2}pR\tilde{B}+\frac{1}{2}R^{-1}\dot{R}^2\tilde{B}-\frac{1}{6}\rho R^3\tilde{A}+\frac{3}{2}R\dot{R}^2\tilde{A}=0 \tag{12}$$
$$-p\tilde{B}-2\ddot{\tilde{B}}-R^{-2}\dot{R}^2\tilde{B}+2R^{-1}\ddot{R}\tilde{B}+2R^{-1}\dot{R}\dot{\tilde{B}}+\rho R^2\tilde{A}+\dot{R}^2\tilde{A}+2R\dot{R}\dot{\tilde{A}}+2R\tilde{A}\ddot{R}=0 \tag{13}$$

Einstein's equations are:

$$\frac{3\left(\frac{d}{dt}R\right)^2}{R^2}=\kappa\rho \quad , \; 2\,R\left(\frac{d^2}{dt^2}R\right)+\left(\frac{d}{dt}R\right)^2=-\kappa R^2 p$$

We use the state equation:

$$p=w\rho$$

to get, for $w\neq -1$:

$$R=R_0 t^{\frac{2}{3(1+w)}}, \qquad \tilde{A}=3wl_2' t^{\left(\frac{w-1}{w+1}\right)}, \tag{14}$$

$$\tilde{B}=R_0^2\; l_2'\; t^b, \qquad b=\frac{4}{3\,w+3}+\frac{w-1}{w+1}$$

l_2':arbitrary constant

Red Shift

Photons moves on a null geodesic of $\mathfrak{g}$:

$$0 = -(1+\kappa_2'\,\tilde{A})dt^2 + (R^2+\kappa_2'\tilde{B})(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2) \tag{15}$$

Introduce:

$$\tilde{R}(t) = \sqrt{\frac{R^2+\kappa_2'\tilde{B}}{1+\kappa_2'\tilde{A}}}(t)$$

Then:

$$z = \frac{\tilde{R}(t_0)}{\tilde{R}(t_1)} - 1 \tag{16}$$

To make the usual connection between redshift and the scale factor, we consider light waves traveling to $r=0$, from $r=r_1$, along the r direction with fixed θ, ϕ. Photons moves on a null geodesic of $\mathfrak{g}$:

$$0 = -(1+\kappa_2'\,\tilde{A})\,d\,t^2 + (R^2+\kappa_2'\,\tilde{B})\,(d\,r^2 + r^2\,d\,\theta^2 + r^2\sin^2\theta\,d\,\phi^2) \tag{17}$$

So,

$$\int_{t_1}^{t_0} d\,t\,\sqrt{\frac{1+\kappa_2'\,t\,A}{R^2+\kappa_2'\,t\,B}} = r_1 \tag{18}$$

A typical galaxy will have fixed r_1, θ_1, ϕ_1. If a second wave crest is emitted at $t = t_1 + \delta t_1$ from $r = r_1$, it will reach $r = 0$ at $t_0 + \delta t_0$, where

$$\int_{t_1+\delta t_1}^{t_0+\delta t_0} dt \sqrt{\frac{1+\kappa_2' t A}{R^2+\kappa_2' t B}} = r_1$$

Therefore, for $\delta t_1, \delta t_0$ small, which is appropiate for light waves, we have:

$$\delta t_0 \sqrt{\frac{1+\kappa_2' t A}{R^2+\kappa_2' t B}}(t_0) = \delta t_1 \sqrt{\frac{1+\kappa_2' t A}{R^2+\kappa_2' t B}}(t_1) \tag{19}$$

Introduce:

$$\tilde{R}(t) = \sqrt{\frac{R^2+\kappa_2' t B}{1+\kappa_2' t A}}(t)$$

We get: $\frac{\delta t_0}{\delta t_1} = \frac{\tilde{R}(t_0)}{\tilde{R}(t_1)}$. A crucial point is that, according to equation (8), δt measure the change in proper time. That is: $\frac{\nu_1}{\nu_0} = \frac{\tilde{R}(t_0)}{\tilde{R}(t_1)}$, where ν_0 is the light frequency detected at $r = 0$ corresponding to a source emission at frequency ν_1. Or in terms of the redshift parameter z, defined as the fractional increase of the wavelength λ:

$$z = \frac{\tilde{R}(t_0)}{\tilde{R}(t_1)} - 1 = \frac{\lambda_0 - \lambda_1}{\lambda_1} \tag{20}$$

We see that $\tilde{R}$ replaces the usual scale factor R in the computation of z.

Distances

Luminosity distance:

$$d_L = \frac{\tilde{R}(t_0)}{\tilde{R}(t_1)} \int_0^z \frac{dz}{H(z)} = (1+z) \int_0^z \frac{dz'}{\tilde{H}(z')} \,, \tilde{H} = \frac{\dot{\tilde{R}}}{\tilde{R}} \tag{21}$$

Let us consider a mirror of radius b that is receiving light from a distant source. The photons that reach the mirror are inside a cone of half-angle ε with origin at the source.

Let us compute ε.The light path of rays coming from a far away source at $\vec{x}_1$ is given by $\vec{x}\,(\rho) = \rho\hat{n} + \vec{x}_1$, $\rho > 0$ is a parameter and $\hat{n}$ is the direction of the light ray.The path reaches us at $\vec{x} = 0$ for $\rho = |\vec{x}_1| = r_1$. So $\hat{n} = -\hat{x}_1 + \vec{\varepsilon}$. Since $\hat{n}\,, \hat{x}_1$ have modulus 1, $\varepsilon = |\vec{\varepsilon}\,| < <1$ is precisely the angle between $-\vec{x}_1$ and $\hat{n}$ at the source.The impact parameter is the proper distance of the path from the origin, when $\rho = |\vec{x}_1|$. The proper distance is determined by the 3-dimensional metric (9). That is $b = \tilde{R}(t_0)\, r_1\, \theta = \tilde{R}(t_0)\, r_1\, \varepsilon$, i.e. $\varepsilon = \frac{b}{\tilde{R}(t_0)\, r_1}$.

Then the solid angle of the cone is $\pi\, \varepsilon^2 = \frac{A}{r_1^2\, \tilde{R}(t_0)^2}$, where $A = \pi\, b^2$ is the proper area of the mirror.The fraction of all isotropically emitted photons that reach the mirror is $f = \frac{A}{4\,\pi\, r_1^2\, \tilde{R}(t_0)^2}$. Each photon carries an energy $h\,\nu_1$ at the source and $h\,\nu_0$ at the mirror. Photons emitted at intervals $\delta\, t_1$ will arrive at intervals $\delta\, t_0$. We have $\frac{\nu_1}{\nu_0} = \frac{\tilde{R}(t_0)}{\tilde{R}(t_1)}, \frac{\delta\, t_0}{\delta\, t_1} = \frac{\tilde{R}(t_0)}{\tilde{R}(t_1)}$. Therefore the power at the mirror is $P_0 = L\frac{\tilde{R}(t_1)^2}{\tilde{R}(t_0)^2}\, f$, where L is the luminosity of the source. The apparent luminosity is $l = \frac{P_0}{A} = L\frac{\tilde{R}(t_1)^2}{\tilde{R}(t_0)^2}\, \frac{1}{4\,\pi\, r_1^2\, \tilde{R}(t_0)^2}$. In Euclidean space, the luminosity decreases with distance d according to $l = \frac{L}{4\,\pi\, d^2}$.This permits to define the luminosity distance:$d_L = \sqrt{\frac{L}{4\,\pi\, l}} = \tilde{R}(t_0)^2 \frac{r_1}{\tilde{R}(t_1)}$. Using (18) we can write this in terms of the red shift:

$$d_L = (1+z) \int_0^z \frac{d\, z'}{\tilde{H}\,(z')}, \tilde{H} = \frac{\dot{\tilde{R}}}{\tilde{R}} \tag{22}$$

Supernova Ia data

The supernova Ia data gives, m (apparent or effective magnitude) as a function of z. This is related to distance d_L by:

$$m = M + 5log(\frac{d_L}{10pc})$$

Here M is common to all supernova and m changes with d_L alone.

We compare δ gravity to General Relativity(GR) with a cosmological constant:

$$H^2 = H_0^2(\Omega_m(1+z)^3 + (1-\Omega_m)), \Omega_\Lambda = 1 - \Omega_m$$

Notice that $\tilde{A} = 0$ for $w = 0$ in (14). So, it seems that we cannot fit the supernova data. However $w = 0$ is not the only component of the Universe. The massless particles that decoupled earlier still remain. It means that true $0 \leqslant w < \frac{1}{3}$, but very close to $w = 0$. So, we will fit the data with $w = 0.1, 0.01, 0.001$ and see how sensitive the predictions are to the value of w.

Using data from Essence, we notice that R^2 test does not change much with w. Moreover l_2 scales with w, $l_2 \sim \frac{a}{3w}$. As an approximation to the limit $w = 0$, we get:

$$\tilde{R}(t) = R(t)\frac{\sqrt{a}}{\sqrt{a-t}} \tag{23}$$

$\sqrt{\frac{1}{3w}}$renormalizes the derivative of $\tilde{R}$ at $t = 0$. It is not divergent, because for $t \to 0$, $w \to \frac{1}{3}$. a is a free parameter.

Of course, the complete model must include the contribution of normal matter($w = 0$) plus relativistic matter $(w = \frac{1}{3})$. But, at later times, the data should tend to (23).

Let us fit the data to the simple scaling model (23).

We get:

$\Omega_m = 0.22 \pm 0.03, M = 43.29 \pm 0.03$, $\chi^2(per point) = 1.0328$, General Relativity
$a = 2.21 \pm 0.12, M = 43.45 \pm 0.06, \chi^2(per point) = 1.0327$, Delta Gravity

δ-gravity with non-relativistic(NR) matter alone give a fit to the data as good as GR with NR matter plus a cosmological constant.

Below, we have the graphs for general relativity and delta gravity versus the data in Essence

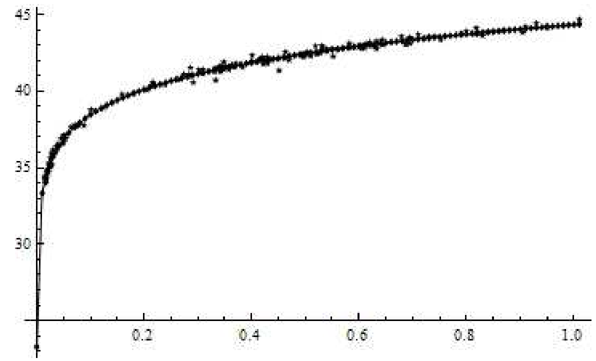

Figure 1. General Relativity with NR matter plus a cosmological constant and Essence data.On the vertical axis, we have m. On the horizontal axis we have z.

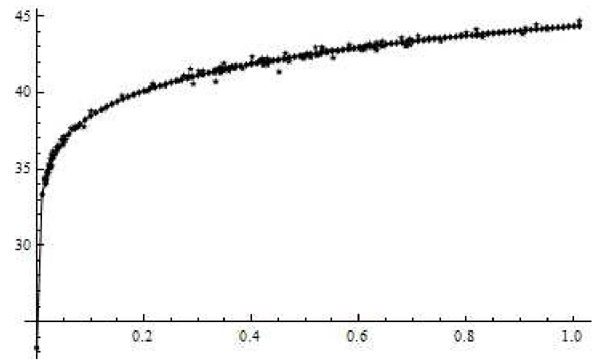

Figure 2. Delta Gravity with NR matter without a cosmological constant and Essence data.On the vertical axis, we have m. On the horizontal axis we have z.

According to the fit to data, a Big Rip will happen at $t = 2.21$ in unities of t_0(today). It is a similar scenario as in R. R. Caldwell, M. Kamionkowski, and N. N.Weinberg, Phys. Rev. Lett.91(2003)071301.

Finally, we want to point out that since for $t \longrightarrow 0$, we have $w \longrightarrow \frac{1}{3}$, then $\tilde{R}(t) = R(t)$. Therefore the accelerated expansion is slower than (23) when we include both matter and radiation in the model.

The Newtonian limit

The motion of a non relativistic particle in a weak static gravitational field is obtained using:

$$g_{\mu\nu} = \begin{pmatrix} -1-2\,U\epsilon & 0 & 0 & 0 \\ 0 & 1-2\,U\epsilon & 0 & 0 \\ 0 & 0 & 1-2\,U\epsilon & 0 \\ 0 & 0 & 0 & 1-2\,U\epsilon \end{pmatrix}$$

which solves Einstein equations to first order in ϵ if:

$$\nabla^2 U = \frac{1}{2}\kappa\rho$$

The solution for $\tilde{g}_{\mu\nu}$ is:

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} \epsilon\,\tilde{U} & 0 & 0 & 0 \\ 0 & 1+\,\epsilon\left(\tilde{U}-2U\right) & 0 & 0 \\ 0 & 0 & 1+\,\epsilon\left(\tilde{U}-2\,U\right) & 0 \\ 0 & 0 & 0 & 1+\epsilon\left(\tilde{U}-2U\right) \end{pmatrix}$$

Solving (3),to first order in ϵ we get:

$$\nabla^2 \tilde{U} = \frac{1}{2}\kappa\rho$$

To recover the Minkowsky metric far from the sources, $\rho\to 0$, we must require there:

$$U\to 0, \tilde{U}\to -\epsilon^{-1}$$

(8) implies :

$$\frac{d^2x^i}{dt^2} = -\phi_{,i}$$
$$\phi = U - \kappa_2'(2U + \tilde{U})$$

That is:

$$\nabla^2\phi = \frac{\kappa}{2}(1-3\kappa_2')\rho, |\kappa_2'| \ll 1$$

The whole effect is a small redefinition of Newton constant.

Gravitational red shift experiments can be used to put bounds on κ_2'. According to (8), the shift in frequency of a source located at x_1, compared to the same source located at x_2 due to the change in gravitational potential is:

$$\frac{\nu_2 - \nu_1}{\nu_1} = \phi_N(x_2) - \phi_N(x_1)$$

where ϕ_N is the usual Newtonian potential, computed with κ as Newton constant. We get;

$$\frac{\Delta\nu}{\nu} = (1 + 2.5 \pm 70 \times 10^{-6})(\varphi_S - \varphi_E + ...)$$

where φ_S is the gravitational potential at the spacecraft position and φ_E is the gravitational potential on Earth. $...$ accounts for additional effects not related to the gravitational potential. We can ascribe the uncertainty of the experiment to κ_2', to get the bound:

$$|\kappa_2'| < 24 \times 10^{-6}$$

This bound is conservative because the Newton constant itself has a larger error (CODATA):

$$G = 6.67428 \pm 0.00067 \times 10^{-11} \frac{m^3}{\mathrm{kgs}^2}$$

Dark Matter

Far from a source the gravitational field correspond to the Schwarzschild solution: pointlike source, spherically symmetric. We get:

The exact solution is:

$$g_{\mu\nu} = \begin{pmatrix} -(1-\frac{a}{r}) & 0 & 0 & 0 \\ 0 & \frac{1}{1-\frac{a}{r}} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 sin(\theta)^2 \end{pmatrix}$$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} -(1-\frac{a}{r}+\frac{ba}{r}) & 0 & 0 & 0 \\ 0 & \frac{1}{1-\frac{a}{r}} - \frac{ab}{r(1-\frac{a}{r})^2} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 sin(\theta)^2 \end{pmatrix}$$

Boundary condition: $g_{\mu\nu} \sim \eta_{\mu\nu}$ $\tilde{g}^{\mu\nu} \sim \eta^{\mu\nu}$ for $r \to \infty$. Notice that still there are 2 arbitrary constants.

Massive Particles

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, h_{00} = \frac{a}{r}, a = 2M$$
$$\tilde{g}_{\mu\nu} = \eta_{\mu\nu} + \tilde{h}_{\mu\nu}, \tilde{h}_{00} = \frac{a(1-b)}{r}, a(1-b) = 2M'$$

The Newtonian potential is:

$$\phi = -((\frac{1}{1+\frac{\kappa_2}{2}'} - \frac{1}{2})h_{00} + \frac{\frac{\kappa_2}{2}'}{1+\frac{\kappa_2}{2}'}\tilde{h}_{00}) = -\frac{M_T}{r}$$

So the total mass of the source is:

$$M_T = M - \frac{\kappa_2' b M}{1+\frac{\kappa_2}{2}'}$$

So, the dark matter mass is:

$$M_{\rm DM} = -\frac{\kappa_2' b M}{1+\frac{\kappa_2}{2}'} \qquad (24)$$

M is the mass coming from the fluid density in Einstein equations. b is a new constant to accomodate DM.

Photons:

The photon trajectory is given by:

$$\left[-(1-\frac{a}{r})-\kappa_2'(1-\frac{a}{r}+\frac{ba}{r})\right]dt^2+\left[\frac{1}{1-\frac{a}{r}}+\kappa_2'(\frac{1}{1-\frac{a}{r}}-\frac{ab}{r(1-\frac{a}{r})^2})\right]dr^2=0$$

$$\left[-1+\frac{1}{r}(a-\frac{\kappa_2' ba}{1+\kappa_2'})\right]dt^2+\left[1+\frac{1}{r}(a-\frac{\kappa_2' ba}{1+\kappa_2'})\right]dr^2=0$$

So, according to photons:

$$M_T=M-\frac{\kappa_2' bM}{1+\kappa_2'}$$

Notice that photons and massive particles see different M_T, but since κ_2' is very small, this difference is hard to detect.

Conclusions and open problems

- Delta Gravity agrees with General Relativity when $T_{\mu\nu} = 0$.
- In a homogeneous and isotropic universe, we get accelerated expansion without a cosmological constant or additional scalar fields.
- Some solutions of Delta Gravity can accomodate Dark Matter. Further research is needed.
- Growth of Density perturbations?
- Anisotropies in the CMBR?

THANK YOU!